Design and Synthesis of High Speed Low Power Signed Digit Adders

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Abstract:

Signed digit (SD) number systems provide the possibility of constant-time addition, where inter-digit carry propagation is eliminated. Such carry-free addition is primarily a three-step process; adding the equally weighted SDs to form the primary sum digits, decomposing the latter to interim sum digits and transfer digits, which commonly belong to {–1, 0, 1}, and finally adding the transfers to the corresponding (i.e., with the same weight) interim sum digits. All the final sum digits are therefore obtained in parallel. The special case of radix-2^h maximally redundant SD number systems is more attractive due to maximum symmetric range (i.e., $[-2^h+1, 2^h-1]$) with only one redundancy bit per SD, and the possibility of more efficient carry-free addition. The previous relevant works use three parallel adders that compute sum and sum±1, where some speed-up is gained at the cost of more area and power. In this paper, we propose an alternative nonspeculative addition scheme that uses carry-save encoding for representation of the primary sum and interim sum digits and computes the transfer digits via a fast combinational logic. The simulation and synthesis of the proposed adder, based on 0.13 μm CMOS technology, shows advantages in terms of speed, power and area.

Keywords: Computer arithmetic, Carry-free addition, Signed-digit number systems, Low power design, Maximal redundancy.

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1. Introduction

Addition is the basic computer arithmetic operation. One study [1] shows that the share of add/subtract operations, among all the floating point arithmetic operations, is 55%. Multiplication, with its 37% share, is the next frequent operation that is often implemented via several add operations.

Traditional ripple-carry adders are very slow due to the long chain of carry-propagation logic. Consequently, the latency of an n-digit carry-ripple adder is linearly depending on n (i.e., O (n)) [2]. Carry-accelerating techniques, for example, in carry look-ahead adders [3] or carry select adders [4] improve the order of latency to O (log n) and O ($\sqrt{n}$), respectively. However, Constant-time (i.e., O (1)) adders are not possible if the sum, as usual, is to be represented in a conventional nonredundant format [5]. Otherwise, carry propagation may be totally eliminated if the sum is allowed to be represented in a redundant format [6].

Signed digit (SD) redundant number systems, where each digit may have a positive or negative value and there is not a single sign bit for the whole number, have been used in several computer arithmetic circuits [7]. SD number systems fall within the generalized signed digit (GSD) number systems that are fixed radix and contiguous number systems with digit set $[-\alpha, \beta]$, where $\alpha, \beta \geq 0$ [6]. A GSD number system is deemed redundant (nonredundant) if the redundancy index $\rho = \alpha + \beta + 1 - r$ is positive (zero), where r is the radix. In the ordinary SD number systems that have balanced digit sets (i.e., $\alpha = \beta$), we have $\rho = 2\alpha+1 - r > 0$. In practice r is a power of two (e.g., 2^h), where the latter inequality leads to $\alpha \geq 2^{h-1}$. The case of $\alpha = 2^{h-1}$ corresponds to the minimally redundant digit set $[-2^{h-1}, 2^{h-1}]$, where $\rho = 1$ and at least h +1 bits are needed to represent a digit. But with the same number of bits α could grow nearly twice up to 2^h-1 corresponding to maximally redundant digit set $[-2^h+1, 2^h-1]$ with $\rho = 2^h - 1$. The latter is particularly attractive due to maximum range of numbers with minimum number of redundancy bits. The higher radix SD number systems [8], where $h \geq 2$, trade-off less storage and data paths for slower arithmetic [9]. For example, the radix-2 (i.e., $h = 1$) SD number system uses two bits to represent each digit in [–1, 1], thus doubling the storage and number of data paths with respect to conventional nonredundant binary systems. However, addition speed is the highest in comparison with the cases with higher radices (i.e., $h \geq 2$).

The main benefit of SD number systems is the possibility of constant time addition, where the latency is small and independent of the number of digits in the operands. A drawback, however, is that conversion of a redundant result to a conventional human readable representation (e.g., nonredundant binary or decimal) may require word wide carry propagation. Therefore, use of redundant representations is justifiable only when there are several arithmetic operations to take place before conversion. This situation occurs in the implementation of composite arithmetic operations, such as multiplication or division with several embedded additions or subtractions, respectively [2] or in floating point addition [10]. It can be found also in whole computations such as function evaluation in general purpose or special purpose hardware units (e.g., digital signal processors [11]). In such applications, the slow final conversion should be well compensated by the latency savings that are compounded over many constant-time redundant operations. Therefore, very fast redundant adders are on demand.

In the addition of any two weighted numbers, the sum digit in position i obviously depends on the two operand digits in the same position. Moreover, for the cases of $\rho = 0$ (i.e., nonredundant), $\rho = 1$, and $\rho \geq 2$, it also depends on all, two, and one less significant digit pairs, respectively. This is further explained below.

- **$\rho = 0$**: In a conventional nonredundant number system the sum digit in each position i (≥ 0) is a function of $2(i+1)$ operand digits; namely two operand digits per each of the positions $i, i-1 \ldots 1, 0$.
- **$\rho = 1$**: For minimally redundant number systems the sum digit in position i (≥ 2) is a function of the digits in positions i, $i-1$ and $i-2$. The constant time addition in this case is called carry-limited [6].
- **$\rho \geq 2$**: In this case, that includes the maximally redundant case of $\rho = r - 1$, the sum digit in each position i (≥ 1) depends on only four operand digits in positions i and $i-1$ [12]. The constant time addition in this case is called carry-free [6].

Each of the three steps of conventional constant time SD addition algorithm (Algorithm 1) is roughly as slow as an h-bit adder. Therefore, it would be desirable to parallelize or fuse these steps for more speed. For example, the case of maximally redundant SD (MRSD) number systems are attractive due to its potential for improvements leading to less latency; noteworthy are the speculative MRSD addition in [8] and [10] that use three parallel h+1-bit adders, per each radix-2^h position, and the nonspeculative approach of [13].

In this paper we present an improved nonspeculative addition scheme for MRSD number systems with power of two radix $r = 2^h$. The paper is organized as follows. The conventional three-step carry-free SD addition algorithm is explained in Section 2. Some previous works on improved maximally redundant SD addition schemes, including a recent one [13], are briefly reviewed in Section 3. The contribution of this paper is presented in Section 4. The results of synthesis and simulation of this work versus three previous contributions, all based on 0.13 μm CMOS technology, are reported in Section 5. Finally we draw our conclusions in Section 6.

2. SD Carry-Free Addition Algorithm

In the conventional nonredundant number systems, cardinality ξ of the digit set is equal to the radix r (e.g., the binary digit set [0, 1] or decimal [0, 9]). The

conventional radix-complement or diminished-radix-complement number systems notwithstanding, in order to allow negative numbers, one could think of digit sets with signed values (e.g., nonredundant radix-10 digit set [–5, 4] and radix-16 digit set [–8, 7]). To build up a balanced range number system, however, it is necessary to allow $\xi > r$ for even radices (e.g., binary, decimal, or hexadecimal digit sets [–1, 1], [–5, 5], or [–8, 8], respectively). The positive redundancy index $\rho = \xi - r > 0$, in such cases leads to redundant number systems, where some values may be redundantly represented by more than one digit combination.

Example 1 (decimal redundancy): Consider the decimal digit set [–7, 7], where $\xi = 15 > r = 10$. In this redundant decimal number system, 2008 may be also represented as 201(–2), where 2008 = 2×1000 + 1×10 + (–2)×1, or 1979 represented also as 20(–2)(–1).

The signed digit (SD) number systems, introduced by Avezienis [14], represent a special case of redundant number systems, where the radix-r digit set [–α, α] is balanced. The constraint $\alpha \geq r/2$ is required for continuity of the represented numbers. This leads to $\xi = 2\alpha + 1 > r$. The most useful property of redundant number systems is the possibility of carry-free addition, where the carry propagation chain is limited to a few number of digits [6].

This chain is only one digit long for SD numbers in case of $r \geq 3$ and $\alpha \geq (r+1)/2$ [14], where the carry generated in any position i will not propagate beyond position $i + 1$. A general carry-free addition scheme for radix-2^h SD number systems with digit set [–α, α], is described by Algorithm 1, where $\alpha > 2^{h-1}$. Fig. 1 depicts a block-diagram representation of the Algorithm.

Algorithm 1 (Carry-free SD addition):

Input: Two n-digit radix-2^h SD numbers $X = x_{n-1}\ldots x_0$ and $Y = y_{n-1}\ldots y_0$, where $-\alpha \leq x_i, y_i \leq \alpha$.

Output: An $(n+1)$-digit radix-2^h SD number $S = s_n\ldots s_0$

I. Compute the n-digit radix-2^h SD number $P = p_{n-1}\ldots p_0 = X + Y$, by digit-parallel computation of $p_i = x_i + y_i$ for $0 \leq i \leq n-1$, where $-2\alpha \leq p_i \leq 2\alpha$.
II. Decompose p_i to transfer t_i+1 and interim sum w_i, for $0 \leq i \leq n-1$, such that $-\alpha + 1 \leq w_i \leq \alpha - 1$, $p_i = w_i + 2^h \times t_{i+1}$, and $t_{i+1} = -1$, 0, and 1 for $p_i \leq -\alpha$, $-\alpha < p_i < \alpha$, and $p_i \geq \alpha$, respectively.
III. Form $s_i = w_i + t_i$, for $0 \leq i \leq n-1$, and set $s_n = t_n$. No new transfer will be generated in this step.

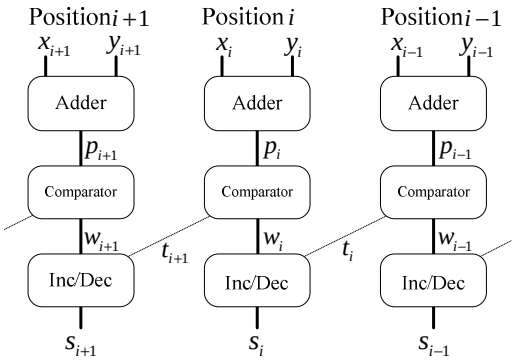

Fig. 1. The three steps of SD addition (Algorithm 1)

Each of the Steps I and III, of Algorithm 1, involves an h-bit addition and Step II requires an h-bit comparison whose complexity is, in general, in the same order as that of an h-bit addition. It would be desirable to reduce the overall latency roughly to that of two or one h-bit addition. The representation or encoding of signed digits is greatly influential on the latency of SD addition. Two's complement encoding of signed digits is believed to lead to the most efficient signed digit addition schemes [8], and is the encoding of choice in all the four designs studied in this paper.

Example 2 (Decimal Carry-free addition): Consider the decimal SD digit set [–7, 7], where $\xi = 15 > r = 10$. Fig. 2 illustrates the application of Algorithm 1 on two 4-digit decimal SD numbers.

	i	4	3	2	1	0
	x_i		2	3	−5	4
	y_i		5	6	−6	2
Step I	p_i		7	9	−11	6
Step II	w_i		−3	−1	−1	6
	t_i	1	1	−1	0	
Step III	s_i	1	−2	−2	−1	6

Fig. 2. A decimal SD addition

3. Efficient SD Addition Schemes

We address three previous efficient MRSD addition schemes numbered as a. (Fig. 3), b. (Fig. 4), and c. (Fig.6). Steps I and III of Algorithm 1 are fused in a. and b., in order to simultaneously compute $p_i - 1$, p_i, and $p_i + 1$. Each scheme, via Step II, derives t_{i+1}, and the three corresponding speculated sum values, in its own way. Note that, in Figs. 3 (5) there are two (one) h+1-bit operations in the critical delay path. The speculative approach, with three parallel adders as in a. and b., is probably the most straight forward approach. However, the nonspeculative scheme c., only computes p_i and simultaneously extracts t_{i+1} directly from x_i and y_i.

a. Fahmy and Flynn [10] have used 2's complement encoding of the MRSD radix-16 number system

to represent redundant digit floating-point numbers with digit set [–15, 15]. The main idea, in their SD adder, is to speculatively compute $p_i - 1$, p_i, and $p_i + 1$ in parallel for all hexadecimal positions, decompose the primary sum digits to $16t_{i+1}$ and the speculative interim sum values $w_i - 1$, w_i, and $w_i + 1$, respectively. The transfer digit t_i selects one of the latter three values as the correct s_i.

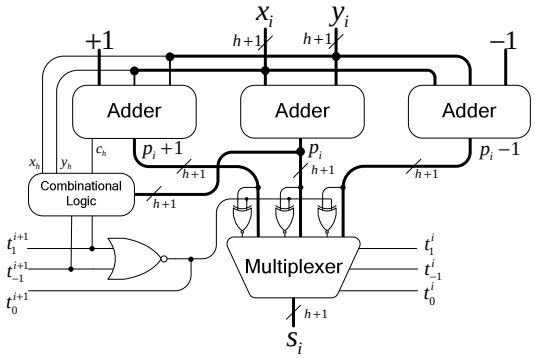

Fig. 3. Position *i* of MRSD adder of [10]

b. The SD addition scheme of [8] is based on an alternative treatment of Step II of Algorithm 1, where p_i is compared to 2^{h-1} instead of α. The corresponding speculative adder architecture is shown by Fig. 4. The rationale for comparison with 2^{h-1} can be understood from Fig. 5, which is primarily showing the valid transfer values based on Algorithm 1 for $\alpha \leq 2^h - 1$. This figure also shows two overlapping regions where either of the two corresponding values, whether on the dashed parts or solid parts, can be assumed by the transfer digit. However, the method in [8] only uses the dashed lines for transfer assignment, which is actually a translation of comparison of p_i with 2^{h-1}. This comparison can be done in constant time independent of the value of h.

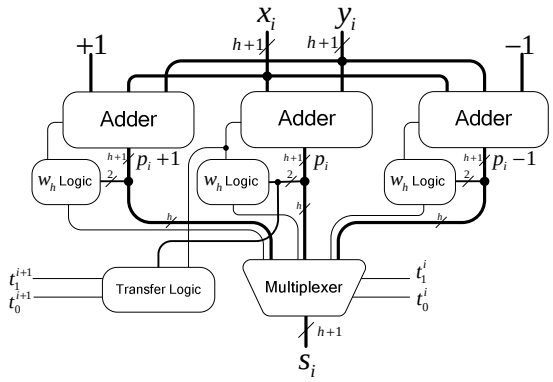

Fig. 4. Position *i* of MRSD adder of [8]

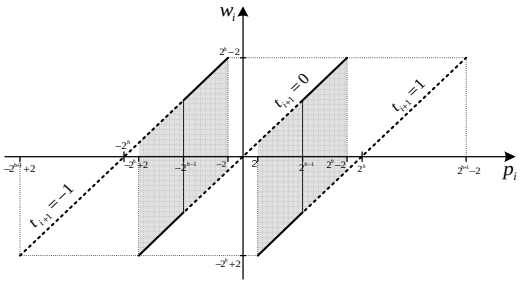

Fig. 5. The overlapping regions of valid values for t_{i+1}

c. The interim sum w_i and the transfer t_{i+1} may be expressed directly as functions of x_i and y_i. However, these functions are hard and inefficient to implement even for moderate values of h (e.g., eight-input functions for $h = 4$). It has been shown in [13] that t_{i+1} can generally be defined as a function of just the most significant bits of x_i and y_i, except for few cases that may be detected by a moderately simple combinational logic. This architecture is depicted in Fig. 6, where the operation of the lower adder starts as soon as the transfer t_i is available at a time that is considerably in advance of completion of operation of the upper adder.

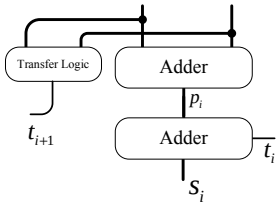

Fig. 6. The nonspeculative SD addition

In the next section, we follow scheme c, but with some simplifications that lead to further improvements in latency, power dissipation and layout area.

4. Improved SD Addition Scheme

Algorithm 1 requires, as the first step, the actual addition $x_i + y_i$. However, one may consider x_i and y_i as the two components of a carry-save two's complement encoding of p_i, which is a special case of weighted bit-set (WBS) encoding [15]. It is illustrated, via symbolic/dot notation, in Fig. 7, where posibits (i.e., normal bits) and negabits (i.e., negatively weighted bits) are represented by lowercase letters inside black dots and upper case letters inside white dots, respectively. With this encoding Step I is bypassed.

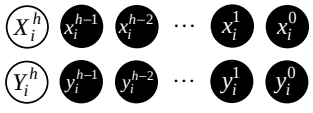

Fig. 7. Carry-save two's complement representation of the position sum p_i

The two negabits X_i^h and Y_i^h weigh 2^h, and as such may directly contribute to the value of the transfer t_{i+1}, whose weight is also 2^h. In fact, if we somehow mange to have one posibit and one negabit in position h, the bit-pair may collectively represent a valid t_{i+1} in [–1, 1]. To arrange this, observe that arithmetic value of the bit collection $\{ X_i^h,\ x_i^{h-1},\ y_i^{h-1} \}$, with respect to position h–1, falls within [–2, 2]. The same range of values may be represented by an equivalent collection of a posibit in position h and two negabits in position h – 1, as shown in Fig. 8. Table I shows the details of this transformation, where the target posibit and negabits are represented by primed variables and it is easily seen that $x_i^{h\prime} = \overline{X_i^h}$, $X_i^{h-1\prime} = \overline{x_i^{h-1}}$, and $Y_i^{h-1\prime} = \overline{y_i^{h-1}}$.

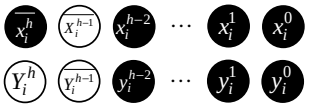

Fig. 8. Equivalent representation of position sum p_i via the transformation of Table I

Table. 1. Justification of transformation from Fig.7 to Fig.8

X_i^h	x_i^{h-1}	y_i^{h-1}	Value	$x_i^{h\prime}$	$X_i^{h-1\prime}$	$Y_i^{h-1\prime}$
0	0	0	0	1	1	1
0	0	1	1	1	1	0
0	1	0	1	1	0	1
0	1	1	2	1	0	0
1	0	0	–2	0	1	1
1	0	1	–1	0	1	0
1	1	0	–1	0	0	1
1	1	1	0	0	0	0

To extract the transfer value, let $\hat{x}_i = \overline{X_i^{h-1}} x_i^{h-2} \cdots x_i^1 x_i^0$ and $\hat{y}_i = \overline{Y_i^{h-1}} y_i^{h-2} \cdots y_i^1 y_i^0$. Then an immediate conclusion of the arrangement (i.e., bit partitioning) of Fig. 8 would be $\hat{t}_{i+1} = \overline{x_i^h} - Y_i^h$ and $\hat{w}_i = \hat{x}_i + \hat{y}_i$. Unfortunately however, it turns out that $\hat{t}_{i+1}$ and $\hat{w}_i$ are not always correct. The exceptions, which can be recognized by a flag $\varphi_i = \overline{x_i^{h-1} \vee \cdots \vee x_i^1} \wedge \overline{y_i^{h-1} \vee \cdots \vee y_i^1} \wedge \overline{x_i^0 \wedge y_i^0}$, and the correct t_{i+1} and w_i are listed in Table II. The transfer $\hat{t}_{i+1}$ is corrected by simply subtracting 1 in all the cases that $\varphi_i = 1$ or $t_{i+1} = \hat{t}_{i+1} - \varphi_i$. This leads to the following equations.

$$\underline{x_i^h} = \overline{Y_i^h \wedge x_i^h \vee \varphi_i}, \qquad \underline{Y_i^h} = Y_i^h \vee x_i^h \qquad (1)$$

Similarly, the interim sum $\hat{w}_i$ gets corrected by adding 2^h. This may be effectively done by adding 1 to both $\overline{X_i^{h-1}}$ and $\overline{Y_i^{h-1}}$ in case of $\varphi_i = 1$. Note that this is always possible, since the value of the latter two negabits in all the correction cases are –1. This leads to Eqns. 2 and 3, respectively, where $\underline{X_i^{h-1}}$ and $\underline{Y_i^{h-1}}$ are the sign bits in the carry-save two's complement representation of w_i.

$$\underline{X_i^{h-1}} = \begin{cases} -1+1=0 & \text{if } \varphi_i = 1 \\ \overline{X_i^{h-1}} & \text{else} \end{cases} \Rightarrow \underline{X_i^{h-1}} = \overline{X_i^{h-1} \vee \varphi_i} \qquad (2)$$

$$\underline{Y_i^{h-1}} = \overline{Y_i^{h-1} \vee \varphi_i} \qquad (3)$$

The transformation from Fig. 7 to Fig. 8 and the latter corrections (i.e., Eqns. 1, 2 and 3) are collectively shown in the first three parts of Fig. 9. The overall delay up to this point is the delay of flag φ_i and a two-input gate in Eqns. 1 to 3. However, since the first (h–1) bits of x_i and y_i remain intact, one may start adding them at time 0 (i.e, when computation of φ_i begins) to compute the first (h – 1) bits of w_i. The carry out of position h–2, a posibit, and the two negabits in position h–1 feed the full adder in that position. For proper functioning of this full adder its two negabit inputs and the negabit carry-out should be inverted [16]. The result is shown in the first row of part 4 of Fig. 9. Recalling Eqn. 1, the two most significant bits of part 3 are extracted to form t_{i+1}. Moreover, to prepare for the last step, the transfer from position i (i.e., t_i) is converted to 2's complement number $T_i^h t_i^{h-1} \cdots t_i^0$ using the logic of Fig. 10.

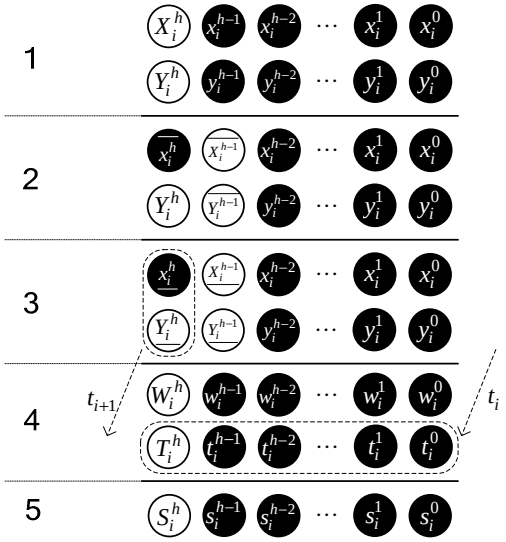

Fig. 9. Digit slice of SD addition in position i

Table. 2. The exceptions for easy extraction of transfer and the corrections

X_i^h	Y_i^h	Range of x_i and y_i	$\hat{t}_{i+1}$	$\hat{w}_i$	Exceptions for (x_i, y_i) pair	Correction	
						t_{i+1}	w_i
0	0	$x_i \geq 0, y_i \geq 0$	1	$-2^h + p_i$	(0, 0), (0, 1), and (1, 0)	0	p_i
0	1	$x_i \geq 0, y_i < 0$	0	p_i	$(0, -2^h+1)$	–1	$2^h + p_i$
1	0	$x_i < 0, y_i \geq 0$	0	p_i	$(-2^h+1, 0)$	–1	$2^h + p_i$
1	1	$x_i < 0, y_i < 0$	–1	$2^h + p_i$	None	None	None

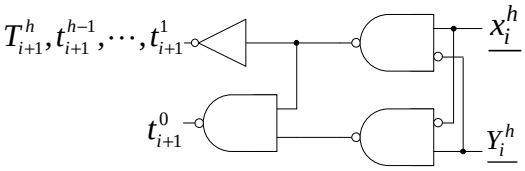

Fig. 10. Conversion of the transfer digit to an equivalent *h*+1-bit two's complement number

The last part of Fig. 9 is an illustration of Step III of Algorithm 1, which may be implemented using an *h*+1-bit two's complement adder. However, given that no new carry would be generated in this step, Eqn. 4 rules the most significant bit of the result, where c_i^h is the carry into position *h*.

$$S_i^h = \overline{c_i^h} \wedge (W_i^h \vee T_i^h) \vee W_i^h \wedge T_i^h \qquad (4)$$

Fig. 11 depicts a digit slice of the overall SD adder based on Fig. 9, where the bold line is the critical delay path. However, the two full adder chains may be replaced by carry look-ahead (CLA) logic, as shown in Fig. 12, in order to reduce latency. The required CLA to replace the lower full adder chain is a simplified one, for the bits of one of the operands are all the same (i.e., $T_i^h = t_i^{h-1} = \cdots = t_i^1 = t_i$ as shown in Fig. 10). This leads to Eqn. 5, where c_i^1 is the carry-into position 1 of the i^{th} digit. For large *h* (e.g., $h = 4k$, $k \geq 2$), a CLA tree with simplified group-generate and group-propagate signals may be used. Eqn. set 6 provides simplified equations for sum bits of digit slice *i* of the SD adder for $h = 4$, where $a = \underline{x_{i-1}^4} \wedge \overline{\underline{Y_{i-1}^4}}$ and $b = \overline{\underline{x_{i-1}^4}} \wedge \underline{Y_{i-1}^4}$. A regular implementation of these equations is depicted by Fig. 13, where the lower half adder in position zero of Fig. 11 and the logic of Fig. 10 are fused for further efficiency. However, in the actual synthesis, gates with higher fan-in may be used.

$$c_i^k = t_i \sum_{j=1}^{k} w_i^j + (t_i + \prod_{j=1}^{k} w_i^j) c_i^1 \qquad (5)$$

$$s_i^0 = w_i^0 \oplus (a \vee b)$$

$$s_i^1 = w_i^1 \oplus \left(a \wedge \overline{w_i^0} \vee b \wedge w_i^0\right)$$

$$s_i^2 = w_i^2 \oplus \left(a \wedge \overline{w_i^1} \wedge \overline{w_i^0} \vee b \wedge w_i^1 \wedge w_i^0\right),$$

$$s_i^3 = w_i^3 \oplus \left(a \wedge \overline{w_i^2} \wedge \overline{w_i^1} \wedge \overline{w_i^0} \vee b \wedge w_i^2 \wedge w_i^1 \wedge w_i^0\right),$$

$$S_i^4 = a \wedge \overline{w_i^3} \wedge \overline{w_i^2} \wedge \overline{w_i^1} \wedge \overline{w_i^0} \vee \overline{b \wedge w_i^3 \wedge w_i^2 \wedge w_i^1 \wedge w_i^0} \wedge W_i^4 . \qquad (6)$$

5. Synthesis and Simulation Results

SD adders operate in a digit-parallel manner. Therefore, for comparison sake, synthesis and simulation of one digit-slice of the SD adder leads to reasonable performance measures for the whole adder. The SD adder of Fig. 11, as modified based on Fig. 12, has been checked for correctness by exhaustive test via VHDL code of one digit-slice. We consider, for the sake of comparison, three previous relevant designs as reference, namely [10], [8], [13]. Besides the delay improvement discussed earlier, considerable area/power improvement with respect to [10] and [8] is expected due to less active hardware redundancy. Some moderate improvement is also expected with reference to [13] due to less complexity of transfer logic and simplified CLA logic.

To confirm the latter expectations, all the four designs were synthesized by Synopsis Design Compiler based on TSMC 0.13 μm CMOS technology. In this endeavour, with the goal of maximum possible speed, we looked for the maximum time constraint that could not be met by any of the four synthesized designs. This was found to be 0.4 ns. The results are compared in Table III, where the 34% less PDP (i.e., product of delay and power) of the proposed design with respect to the best previous one is quite noticeable.

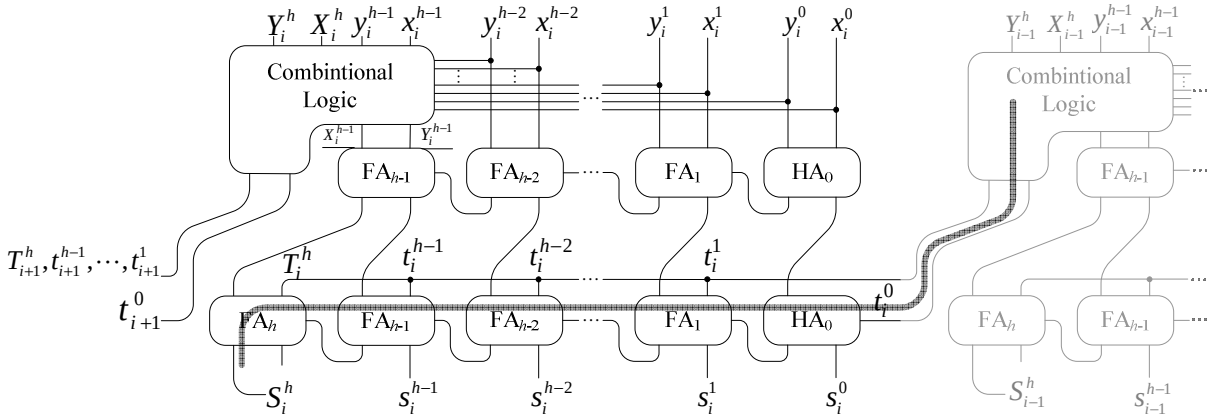

Fig. 11. Digit slice of SD adder based on Fig. 9

Table. 3. Simulation results for single digit MRSD adders with h = 4 based on 0.13 µm CMOS

Design reference	Delay (ns)	Power dissipation		Area(μm^2)	Delay × power
		Dynamic (mW)	Static (pW)		
[10]	0.61	1.95	36.53	2255.7	1.19
[8]	0.57	2.19	40.64	2473.8	1.25
[13]	0.50	1.98	41.14	2480.5	0.99
New MRSD Adder	0.46	1.42	22.01	1707.9	0.65

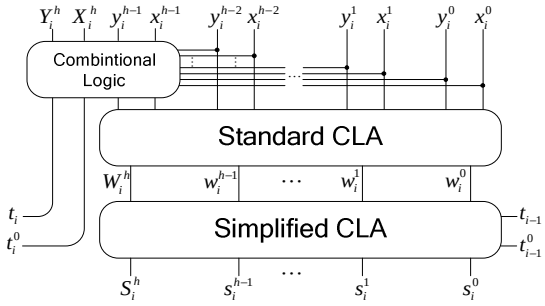

Fig. 12. The SD adder with CLA components

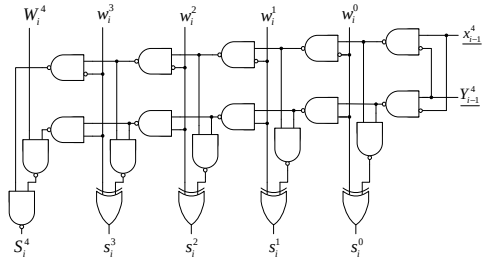

Fig. 13. The simplified CLA logic for h = 4 replacing the lower FA-chain of Fig. 11

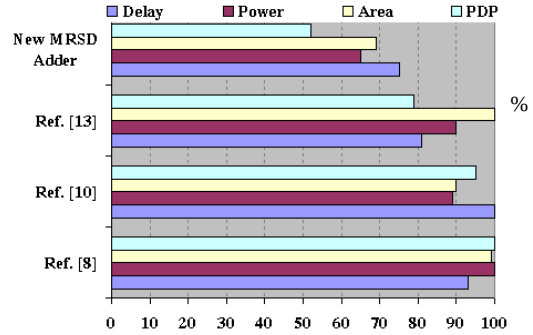

Fig. 14. Performance comparison between the new MRSD adder and three previous ones.

6. Conclusions

We examined three previous efficient implementations of maximally redundant signed digit adders. Then, we proposed a new MRSD addition scheme based on carry-save two's complement encoding of primary sum-digits that are readily available by simply aligning the equally weighted digits of the operands. The first step of conventional SD addition algorithm is thus bypassed. The primary sum digits are partitioned to a transfer part and an interim sum. Whereas this simple partitioning leads to invalid results in few exceptional cases of the operands, a flag is computed to indicate the exceptions and to enforce corrections. Finally, the last step of conventional SD addition (i.e., adding the interim sum digits with the transfer coming from the next less significant digit position) is performed by a simplified carry look-ahead logic.

The new MRSD adder is checked for correctness via exhaustive tests based on VHDL code describing the adder. Synthesis and simulation of the proposed adder shows better performance in terms of delay, power dissipation and layout area in comparison with three previous contributions. Fig. 14, based on the results tabulated in Table III, depicts these advantages.

Research is on going for further performance improvement in SD adders, and use of them in more sophisticated hardware units such as multiplication, division, and floating-point arithmetic circuits. The through benefit of the proposed adder can be better evaluated in realistic benchmark applications of such complex hardware units.

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References

[1] Oberman S. F., Design Issues in High Performance Floating Point Arithmetic Units. PhD thesis, Stanford University, Nov. 1996.

[2] Parhami B., Computer Arithmetic: Algorithms and Hardware Designs, Oxford, 2000.

[3] Doran R.W., "Variants of an Improved Carry Look-Ahead Adder," *IEEE Trans. Computer*, vol. 37, no. 9, pp. 1110-1113, 1988.

[4] Sklansky J., "Conditional-Sum Addition Logic," *IRE Trans. Elec*. Comp. vol. 9, no. 2, pp. 226-231, 1960.

[5] Winograd S., Watson T. J., Heights Y., "On the Time Required to Perform Addition," *Journal of the ACM* (JACM), vol. 12, no. 2, pp. 277 – 285, 1965.

[6] Parhami B., "Generalized Signed-Digit Number System: A Unifying Framework for Redundant Number Representation," *IEEE Trans. on Computer*, Vol. 39, No. 1, pp. 89-98, 1990.

[7] González Alejandro F., and P. Mazumder, "Redundant arithmetic, algorithms and implementations," *Integration the VLSI Journal*, Vol. 30, Issue 1, pp. 13-53, Nov. 2000.

[8] Jaberipur G., and M. Ghodsi, "High Radix Signed Digit Number Systems: Representation Paradigms," *Scientia Iranica*,Vol. 10, No.4, pp. 383-391, Oct. 2003.

[9] Phatak D. S., and I. Koren, "Hybrid Signed-Digit Number Systems: A Unified Framework for Redundant Number Representations with Bounded Carry Propagation Chains" *IEEE Trans. on Computers*, Vol. 43, No. 8, pp 880-891, Aug. 1994.

[10] Fahmy H., and M.J. Flynn, "The Case for a Redundant Format in Floating-point Arithmetic," *Proc. 16th IEEE Symp. Computer Arithmetic*, pp. 95-102, 2003.
[11] Hewlitt R.M., and E.S. Jr. Swartzlantler, "Canonical signed digit representation for FIR digital filters," *IEEE Workshop on Signal Processing Systems*, pp. 416-426, 2000.
[12] Jaberipur G. and B. Parhami, "Stored-Transfer Representations with Weighted Digit-Set Encodings for Ultrahigh-Speed Arithmetic," *IET Circuits, Devices, and Systems*, Vol. 1, pp. 102-110, Feb. 2007.
[13] Jaberipur G. and S. Gorgin, "A Nonspeculative One-Step Maximally Redundant Signed Digit Adder," Lecture Notes on Computer Science, CSICC 2008, CCIS 6, pp. 235–242, 2008.
[14] Avizienis A., "Signed-digit number representations for fast parallel arithmetic," *IRE Trans. on Electronic Computers*, Vol. EC-10, pp. 389–400, Sep. 1961.
[15] Jaberipur G., B. Parhami, and M. Ghodsi, "Weighted Two-Valued Digit-Set Encodings: Unifying Efficient Hardware Representation Schemes for Redundant Number Systems," *IEEE Trans. Circuits and Systems I*, Vol. 52, No. 7, pp. 1348, 1357, Jul. 2005.
[16] Aoki T., Y. Sawada, and T. Higuchi, "Signed-Weight Arithmetic and Its Application to a Field-Programmable Digital Filter Architecture," *IEICE Trans. Electronics*, Vol. E82-C, No. 9, pp. 1687-1698, Sep. 1999.