A New Method for Considering Distribution Systems in Voltage Stability Studies

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Abstract :

In methods presented to calculate the voltage collapse point, the transmission system is usually the only part of the power system that is completely modeled. Distribution systems are often replaced by aggregate load models because the use of the detailed model of distribution systems in voltage stability analysis not only increases the computation time, but also decreases the probability of convergence of calculations. But this replacement can cause a considerable error in determining voltage collapse point. In this paper, a method for considering distribution systems in voltage stability studies is presented. In this method, a detailed model of distribution systems is used. The loads are connected to the secondary side of distribution transformers. An iterative method is used to calculate the voltage collapse point. In each iteration, first the value of active and reactive power delivered to transmission buses that supply distribution systems is increased. Then for each given value of the delivered power, the power received at the secondary side of distribution transformers is calculated. With this work, the voltage collapse point is determined using separately solving transmission and distribution system equations. On the other word, instead of solving a large set of equations, some smaller sets of equations are solved. The simulation results show the effectiveness of the proposed method.

Keywords: Distribution system, Voltage stability, Static analysis, Dynamic analysis.

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1. Introduction

One of the important requirements in power system operation is to quickly and accurately determine the voltage collapse point. This point is Saddle-Node Bifurcation (SNB), in which with more increase in load powers, there is no any longer equilibrium point for system equations. To determine the SNB point in short-term voltage stability studies, dynamic equations corresponding to fast state variables are replaced by their equilibrium equations and slow state variables are considered as slowly varying parameters [1]. In long-term voltage stability studies, which is the subject of this paper, all dynamic equations are replaced by their equilibrium equations. Load Tap Changing (LTC) transformers play a very important role in long-term voltage stability analysis. They keep the voltage magnitude of the voltage-controlled side constant. This causes the load characteristic seen from the other side of transformer to become of approximately constant power type. This is the reason why the constant power model is used for loads connected to transmission buses. The magnitude of this constant power depends on the consumed power in the distribution system connected to the voltage-controlled side of transformer. After a voltage decrease, for example due to a load increase or line outage, the power system becomes voltage unstable if tap changers can not restore voltage to a predefined set point. This happens when the power system can not deliver the demanded power in the setting voltage value to the transformer location. In radial distribution systems connected to transmission buses, there are usually several cascaded LTC transformers at the EHV/HV and HV/MV substations. The LTC transformers in EHV/HV substations regulating HV-side voltages are upstream and close to the transmission buses while the ones in HV/MV substations regulating MV-side voltages are downstream. The loads seen from EHV and HV buses are of the constant power type until the corresponding tap limits are reached. Voltage stability is maintained if the demanded load power in MV buses can be supplied by the power system. The distance between MV and transmission buses can not be considered while the distribution systems are being replaced by an aggregate load model. So a detailed model of distribution systems is needed.

In almost all of the presented methods for voltage stability analysis, the transmission system is the only part of the power system that is completely modeled. On the other hand, in some papers aimed at voltage control in distribution systems, the transmission system is replaced by a voltage source [2, 4]. The value of voltage stability margin depends on both transmission and distribution systems. There are few papers considering distribution systems in addition to transmission ones in determining voltage stability margin [5-8]. In [5, 6], the necessity for considering distribution systems in voltage stability studies has been illustrated. The analysis of a large power system represented in detailed model for on-line and real-time applications is not possible if no simplifications are made. One of the simplification methods is the use of dynamic equivalents for particular parts of a power system. In these methods, the power system decomposes into study area and one or more external areas. Each external area is replaced by an equivalent model [9-11]. Most equivalencing methods have been presented for transient and small signal stability studies. Few papers focus on system simplification for voltage stability studies [7, 8]. In [7], a method for reduction of a distribution system with cascaded LTC transformer to an impedance in series with a modified LTC transformer has been presented. The impedance is based on the losses in the unreduced distribution system. The time delay and step size of the equivalent LTC transformer are equal to those of the main supply transformer. The sum of the loads in the unreduced system is chosen as the load of the equivalent distribution system. The response of unreduced and equivalent distribution systems to a step change in the supplying voltage has been compared, but the voltage collapse point when using two models has been not determined. In [8], two equivalent models for distribution systems have been presented, equivalent in section (ES) and equivalent in total (ET). The latter is the same as in [7], but the former reduces the distribution system in each voltage level to an equivalent section. In the simulations performed on the IEEE 9-bus test system, a radial distribution system is connected to bus 9 that is a no-load bus. None of aggregate load models have been replaced by a distribution system. To determine the voltage collapse point, the loads in the distribution system are kept constant and one of aggregate loads is increased. This is not the correct way of considering distribution systems in voltage stability studies and can not determine the accuracy of the proposed equalization method.

In this paper, a detailed model of distribution systems without any equalization is used. The maximum value of the loads in the secondary side of distribution transformers is determined. This is done using separately solving transmission and distribution system equations. On the other word, instead of solving a large set of equations, some smaller sets of equations are solved.

2. Proposed Method

2.1. Classification of Voltage Stability Aanalysis Methods

Methods have been proposed in the literature for voltage stability analysis can be classified into measurement-based methods [12-14] and model-based ones [15-17]. The formers are based on the fact that maximum-loading point is detectable using the local measurement of voltage and current phasors. Since the system model is not used, the measurement-based methods are suitable for the analysis of the current power system only, and are unusable for the examination of contingency states. The model-based methods use dynamic and static techniques. The formers are time consuming and unsuitable when the investigation of a wide range of system conditions and a large number of contingencies is required [18]. Static techniques use equilibrium equations especially the power flow equations instead of differential equations. This is the method used in this paper.

It is well known that voltage collapse point coincides with SNB point, in which with more increase in load powers, there is no any longer equilibrium point for system equations. So to find out whether the system is voltage stable for a given condition, it is enough to investigate the existence of equilibrium point. The system equations consist of differential and algebraic ones. To calculate the equilibrium point, the differential equations are replaced by their equilibrium equations. With this work, if the aggregate load model is used for distribution systems, the system equations will be the same as power flow ones. To determine the voltage collapse point, the system load is increased and the equilibrium point is calculated for each load level. This continues up to the point where there is no any longer equilibrium point. The latest equilibrium point is the voltage collapse point.

The generator models employed are the same as used in the conventional power flow equations. Also in distribution system models with cascaded LTC transformers, the voltage-controlled buses are modeled as PQV buses. Because at these buses, in addition to the active and reactive powers, the voltage magnitude is known. In each calculated equilibrium point, if any one of tap values is outside the limit, the tap value is set equal to the limit violated, and the corresponding voltage-controlled bus is treated as a PQ bus. Then a new equilibrium point is calculated. In this paper, the voltage collapse point determined using the static analysis is validated by the dynamic simulation. Since the aim is to study the long term voltage instability, the time response of tap changers is considered only, and dynamic equations corresponding to fast state variables are replaced by their equilibrium equations. This method of simulation is known as Quasi Steady-State (QSS) approximation [19].

2.2. Separately Solving Transmission and Distribution System Equations

To explain the proposed method, the simple system shown Fig. 1 is used. In this figure, a radial distribution system is connected to a two-bus transmission system.

The loads are modeled by the widely used characteristic known as the exponential load model:

$$P=\lambda P_0 V^{\alpha}$$
$$Q=\lambda Q_0 V^{\beta} \tag{1}$$

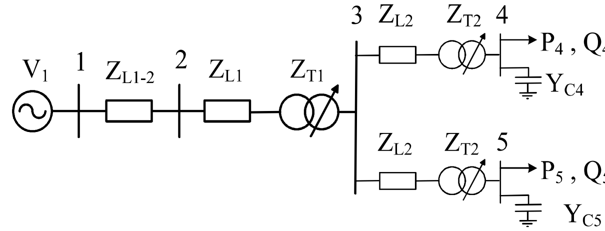


Fig. 1. A simple system.

In this model, λ is a parameter called loading factor, which is used to increase the load demand. Its value is set equal to one for the base load. P_0 (resp. Q_0) is the active (resp. reactive) power consumed in the base load with $V=1$ *pu*. For each given value λ, the system has a stable equilibrium point if the tap changers succeed to restore the voltage magnitude of voltage-controlled buses to their reference values. The voltage collapse point is the maximum value of λ, called λ^*, for which there is an equilibrium point. In the proposed method, to calculate equilibrium points, instead of simultaneously solving transmission and distribution system equations, separately solving them is done. To do this work, the system in Fig. 1 is divided into transmission and distribution systems as shown in Fig. 2. At first it is assumed that the values of P_2 and Q_2 in Fig. 2(a) are equal to the sum of the active and reactive loads in the distribution system in the base load. Then by solving the transmission system equations, the voltage magnitude V_2 is determined. With given known values of P_2 and V_2 from Fig. 2a, the distribution system equations in Fig. 2b are solved to determine the value of λ. In these equations, buses 3, 4 and 5 are PQV buses. The value of Q_2 is assumed to be unknown because it is necessary to match the number of equations to the number of unknowns. After the distribution system equations are solved, the value of Q_2 is determined. The new power factor in the load bus in Fig. 2(a) is based on the values of P_2 and calculated Q_2 from Fig. 2(b). With this power factor, the load is increased in Fig. 2(a) and the transmission system equations are solved again. In each step of the load increase, the power factor is updated from the value of Q_2 obtained from solving the distribution system equations. The steps continue until solving the distribution system equations fails to converge to a solution. The latest calculated value of λ is λ^*.

The distribution system equations are the power-balance equations at buses 2 to 5 in Fig. 2(b) as Eqs. 2 to 8.

$$P_2=\sum_{j=2}^{5} Y_{2j} V_2 V_j \cos(\delta_2-\delta_j-\theta_{2j}) \tag{2}$$

$$P_3 = 0 = \sum_{j=2}^{5} Y_{3j} V_3 V_j \cos(\delta_3 - \delta_j - \theta_{3j}) \quad (3)$$

$$Q_3 = 0 = \sum_{j=2}^{5} Y_{3j} V_3 V_j \sin(\delta_3 - \delta_j - \theta_{3j}) \quad (4)$$

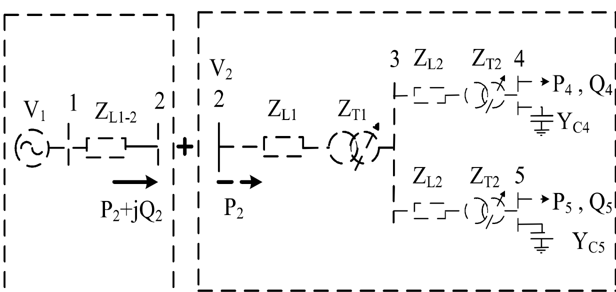
$$-\lambda P_{04} V_4^{\alpha} = \sum_{j=2}^{5} Y_{4j} V_4 V_j \cos(\delta_4 - \delta_j - \theta_{4j}) \quad (5)$$

$$Y_{C4} V_4^2 - \lambda Q_{04} V_4^{\beta} = \sum_{j=2}^{5} Y_{4j} V_4 V_j \sin(\delta_4 - \delta_j - \theta_{4j}) \quad (6)$$

$$-\lambda P_{05} V_5^{\alpha} = \sum_{j=2}^{5} Y_{5j} V_5 V_j \cos(\delta_5 - \delta_j - \theta_{5j}) \quad (7)$$

$$Y_{C5} V_5^2 - \lambda Q_{05} V_5^{\beta} = \sum_{j=2}^{5} Y_{5j} V_5 V_j \sin(\delta_5 - \delta_j - \theta_{5j}) \quad (8)$$

where δ_i and V_i are the voltage angle and voltage magnitude at bus i, respectively. In these equations, the value of λ, the voltage angle at buses 3 to 5 and the tap values are 7 unknown variables. The voltage angle at bus 2 is set equal to zero. Tap changing transformers are represented by the equivalent-π model as shown in Fig. 3. This causes that the entries of Y_{bus} become functions of the tap values. In the equivalent-π model, a is the tap value and Y is the transformer admittance. V_i and V_j are the phasor voltages on the primary and secondary sides, respectively.



a) Transmission system b) Distribution system

Fig. 2.The division of system into transmission and distribution subsystems

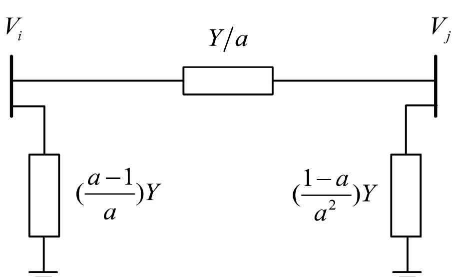


Fig.3. The equivalent-π model.

In a real power system, there is a large number of distribution systems for each of which a different λ^* is obtained. The value of λ^* in each distribution system, in addition to various characteristic of the distribution system, depends on the strength of corresponding transmission bus and the rate of load increase in that bus. With change in the rate of load increase in different buses, there is an infinite number of ways to reach a voltage collapse point. A voltage collapse point is reached when the first divergence occurs in solving distribution systems equations. The flowchart of the proposed method is shown in Fig. 4.

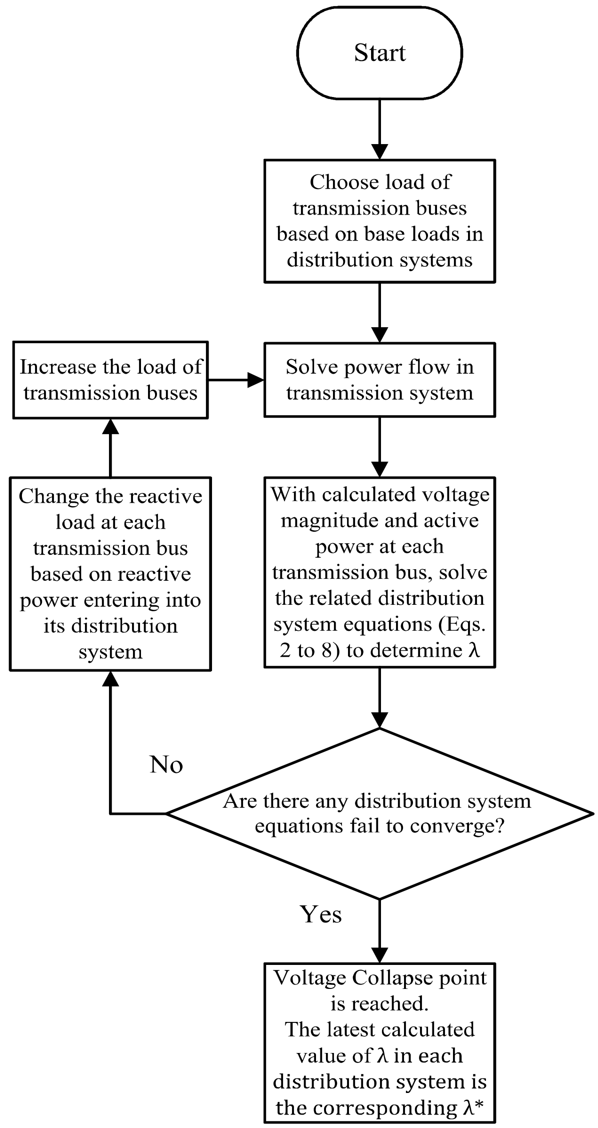


Fig. 4. The flowchart of the proposed method.

3. Test results

To show the effectiveness of the proposed method, the IEEE 9 and 30-bus test systems are used. For the IEEE 9-bus test system, three radial distribution systems with the structure as shown in Fig. 1 are connected to buses 5, 6 and 8 (Fig. 5). The data of these distribution systems is as appendix A. The computer system used in this study is CPU 2700 *MHZ* and 1.99 *GB* of RAM.

In the static analysis, the voltage magnitude of voltage-controlled buses is set equal to 1 *pu*. Three cases based on the different rates of load increase are simulated. In case 1, the load in different distribution systems is increased with the same rate. In case 2, the load in the distribution system being connected to bus 6 is increased 2 and 2.5 times faster than those being connected to buses 5 and 8, respectively. In case 3, the load in the distribution system being connected to bus 8 is increased 2 times faster than those being connected to buses 5 and 6. The voltage collapse points in different cases that have been obtained by different methods are compared in Tables 1 to 3. In these tables, λ_5^*, λ_6^* and λ_8^* are the maximum value of λ in the distribution systems being connected to buses 5, 6 and 8, respectively.

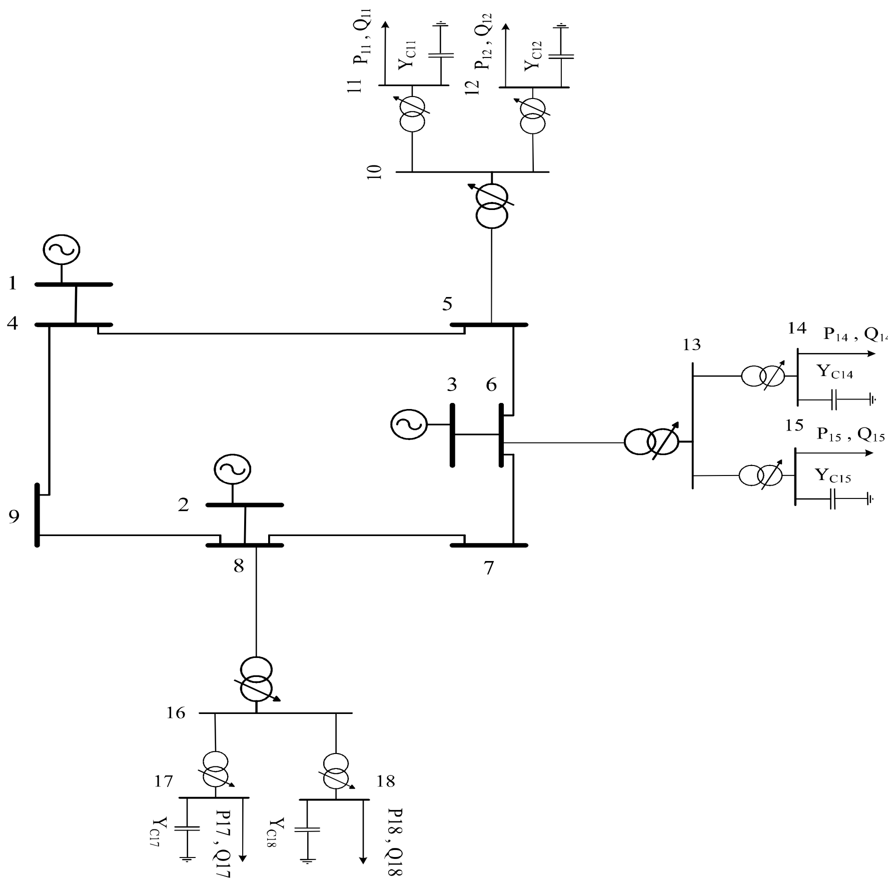


Fig. 5. IEEE 9-bus test system with three radial distribution systems.

In table 1, the distribution system being connected to bus 5 is the first one in which solving its equations fails to converge. These systems in tables 2 and 3 are those being connected to buses 6 and 8, respectively. It is shown that the proposed method with a maximum error of 3%, has reduced the computation time about 45%. The collapse point calculated by the proposed method, even though may be in the infeasible region, can be used as the initial point to exactly determine the voltage collapse point (for example by the method presented in [20]).

Figs. 6 to 8 show the time response of voltage magnitude in buses 11, 19 and 25, respectively when the loading factor is increased from one to the maximum value. Since the aim is to study the long term voltage instability, the time response of tap changers is considered only, and dynamic equations corresponding to fast state variables are replaced by their equilibrium equations.

Table. 1. Comparison of calculated voltage collapse points in case 1 (9-bus test system)

Voltage collapse point (λ^*)	The used method	
	Simultaneously solving equations	The proposed method
λ_5^*	1.880	1.871
λ_6^*	1.880	1. 949
λ_8^*	1.880	1. 915
Computation time (S)	**4.4**	**2.2**

Table. 2. The comparison of calculated voltage collapse points in case 2 (9-bus test system)

Voltage collapse point (λ^*)	The used method	
	Simultaneously solving equations	The proposed method
λ_5^*	1.460	1.448
λ_6^*	3.300	3.322
λ_8^*	1.920	1.936
Computation time (S)	**4.5**	**2.3**

Table. 3. The comparison of calculated voltage collapse points in case 3 (9-bus test system)

Voltage collapse point (λ^*)	The used method	
	Simultaneously solving equations	The proposed method
λ_5^*	1.730	1.744
λ_6^*	1.730	1.787
λ_8^*	2.460	2.476
Computation time (S)	**4.3**	**2.2**

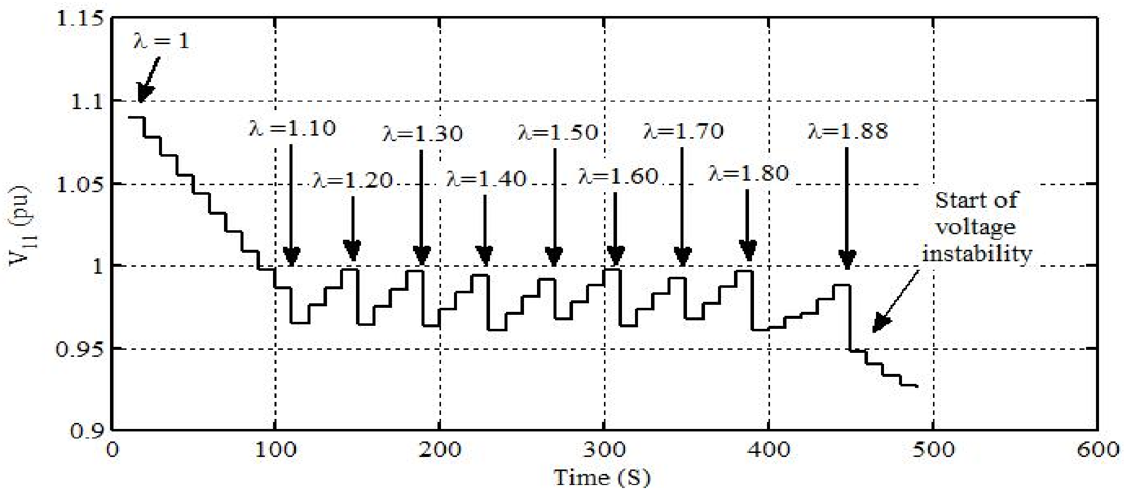


Fig. 6.Time response of voltage magnitude in bus 11 (case 1)

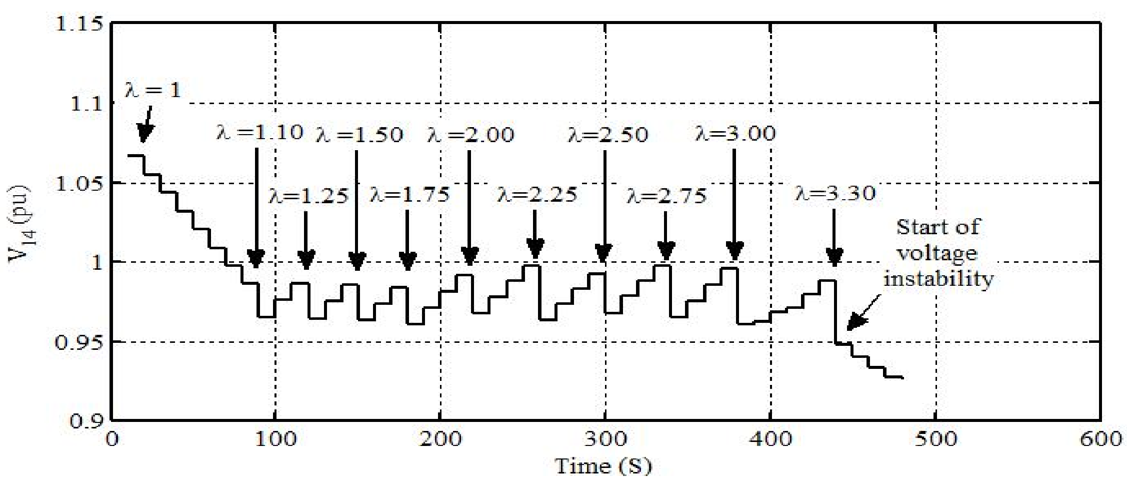


Fig. 7. The time response of voltage magnitude in bus 14 (case 2)

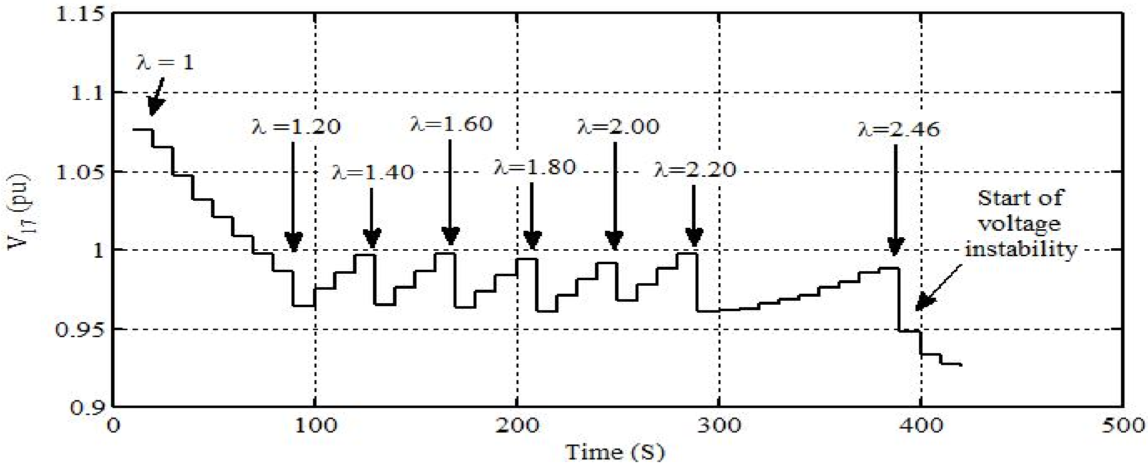


Fig. 8. The time response of voltage magnitude in bus 17 (case 3)

After each tap movement, a power flow is solved and new voltages are determined. The assumed time delay for the tap movement in upstream and downstream transformers is 10 and 20 seconds, respectively. The increase in the loading factor causes a sudden voltage drop at all buses. Before reaching voltage instability, the voltage magnitude at voltage-controlled buses recovers inside their corresponding deadbands (assumed here between 0.98 and 1.02 p.u.) by tap changer operations. When voltage instability is reached, the operation of tap changers results in a further reduction and collapse of voltages. As seen, the voltage collapse in different cases starts when the loading factors are equal to the maximum values that have been calculated by the static analysis.

To simulate the proposed method in the IEEE 30-bus test system, the loads connected to buses 3, 4, 7, 10, 12, 14-21, 23, 24, 26, 29 and 30 are replaced by 18 radial distribution systems with equal structure similar to the one in Fig. 1. The data of these distribution systems is the same as shown in appendix A with capacitive susceptances equal to 0.2 *pu*. Two cases are simulated. In case 1, the load in different distribution systems is increased with the same rate. In case 2, the load in the distribution system being connected to buses 16-21, 23, 24, 26, 29 and 30 is increased 2 times faster than the others. The results obtained by different methods are compared in Tables 4 and 5. It is shown that in case 1 the proposed method with a maximum error of 3%, has reduced the computation time about 55%. This time has been reduced about 64% in case 2. In this case, the maximum error is 4%.

Table. 4. Comparison of calculated voltage collapse points in case 1 (30-bus test system)

Voltage collapse point (λ^*)	The used method: Simultaneously solving equations	The used method: The proposed method
λ_3^*	1.90	1.9583
λ_4^*	1.90	1.9456
λ_7^*	1.90	1.9018
λ_{10}^*	1.90	1.9497
λ_{12}^*	1.90	1.9313
λ_{14}^*	1.90	1.9490
λ_{15}^*	1.90	1.9436
λ_{16}^*	1.90	1.9552
λ_{17}^*	1.90	1.9382
λ_{18}^*	1.90	1.9567
λ_{19}^*	1.90	1.9398
λ_{20}^*	1.90	1.9592
λ_{21}^*	1.90	1.9131
λ_{23}^*	1.90	1.9561
λ_{24}^*	1.90	1.9375
λ_{26}^*	1.90	1.9545
λ_{29}^*	1.90	1.9586
λ_{30}^*	1.90	1.9381
Computation time (S)	**30.1**	**13.4**

4. Conclusions

In the presented method, the voltage collapse point is determined using the detailed model of distribution systems. The method is based on separately solving transmission and distribution system equations. With this work, instead of solving a large set of equations, some smaller sets of equations are solved. The simulations performed on the IEEE 9 and 30-bus test systems show that the proposed method, with a maximum error of 4%, considerably reduce the computation time. This reduction in the computation time is very important when the examination of a wide range of system conditions and a large number of contingencies is needed.

Table. 5. The comparison of calculated voltage collapse points in case 2 (30-bus test system)

Voltage collapse point (λ^*)	The used method: Simultaneously solving equations	The used method: The proposed method
λ_3^*	1.57	1.6366
λ_4^*	1.57	1.6221
λ_7^*	1.57	1.5721
λ_{10}^*	1.57	1.6268
λ_{12}^*	1.57	1.6058
λ_{14}^*	1.57	1.6260
λ_{15}^*	1.57	1.6198
λ_{16}^*	3.14	3.2030
λ_{17}^*	3.14	3.1837
λ_{18}^*	3.14	3.2048
λ_{19}^*	3.14	3.1855
λ_{20}^*	3.14	3.2077
λ_{21}^*	3.14	3.1549
λ_{23}^*	3.14	3.2041
λ_{24}^*	3.14	3.1829
λ_{26}^*	3.14	3.2023
λ_{29}^*	3.14	3.2070
λ_{30}^*	3.14	3.1835
Computation time (S)	**28.6**	**10.3**

Appendix A.

Distribution systems data:

Line impedance:

$Z_{L1} = 0.02 + j0.10\, pu$

$Z_{L2} = 0.01 + j0.02\, pu$

Transformer impedance:

$Z_{T1} = 0.01 + j0.05\, pu$

$Z_{T2} = 0.01 + j0.08\, pu$

Upstream LTC data:

Upper limit of tap value $= 1.12\, pu$

Lower limit of tap value $= 0.88\, pu$

Upper recovery level $= 1.02\, pu$

Lower recovery level $= 0.98\, pu$

Step size $= 0.01\, pu$

Time delay $= 10\ S$

Downstream LTC data:

Upper limit of tap value $= 1.15\, pu$

Lower limit of tap value $= 0.85\, pu$

Upper recovery level $= 1.02\, pu$

Lower recovery level $= 0.98\, pu$

Step size $= 0.01\, pu$

Time delay $= 20\, S$

Capacitive susceptance:

$Y_{C11} = 0.27\, pu$, $Y_{C12} = 0.27\, pu$

$Y_{C14} = 0.10\, pu$, $Y_{C15} = 0.10\, pu$

$Y_{C17} = 0.20\, pu$, $Y_{C18} = 0.20\, pu$

Load parameters:

$\alpha = 1.50\, pu$, $\beta = 2.50\, pu$

$P_{011} = 0.50\, pu$, $P_{012} = 0.40\, pu$

$P_{014} = 0.20\, pu$, $P_{015} = 0.18\, pu$

$P_{017} = 0.40\, pu$, $P_{018} = 0.30\, pu$

$Q_{011} = 0.20\, pu$, $Q_{012} = 0.15\, pu$

$Q_{014} = 0.10\, pu$, $Q_{015} = 0.08\, pu$

$Q_{017} = 0.15\, pu$, $Q_{018} = 0.12\, pu$

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