PMU-Based Matching Pursuit Method for Black-Box Modeling of Synchronous Generator

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Abstract :

This paper presents the application of the matching pursuit method to model synchronous generator. This method is useful for online analysis. In the proposed method, the field voltage is considered as input signal, while the terminal voltage and active power of the generator are output signals. Usually, the difference equation with a second degree polynomial structure is used to estimate the corresponding parameters of the regressor matrix. This matrix contains the elements of the difference equation for all sampling intervals. However, the number of parameters that should be estimated increases significantly with the increase in the difference equation order. In this paper, the matching pursuit method is implemented to identify important parameters with a very good accuracy and in a shorter time. In order to show the effectiveness of the proposed algorithm, it is applied to a nonlinear generator and the required signals are sampled using a phasor measurement unit (PMU). The simulation results show the effectiveness and precision of the proposed method.

Keywords: Matching pursuit, Parameter estimation, Synchronous generator, Phasor measurement unit.

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1. Introduction

In recent years, power engineers have tended to change the traditional power systems into the smart grids. A smart grid is a complex network that uses communication infrastructures for its elements such as generation units, loads, etc. These infrastructures lead to some considerable advantages such as self-healing, power quality, generation and storage accommodation, etc. [1]. The introduction of Phasor Measurement Units (PMUs) has been a revolution in smart grids development. The idea of PMUs was proposed in 1980s and the first PMU was built in 1992 [2-4]. After installation of PMU, it is possible to measure the voltage phasor of the bus bar and current phasors of transmission lines which are connected to it. Usually, PMUs provide 60 samples in a second of the desired signal.

The synchronous generator is a nonlinear system and its nonlinearity can be classified into two types: structured and unstructured. The first type of nonlinearities has a known structure such as rotor angle which is modeled by sine and cosine functions. The second type does not have a specified model such as magnetic saturation. Considering these nonlinearities, getting a reliable model of generators is essential to analyze transient disturbances and have appropriate control on synchronous machines.

Generally, a system can be modeled by three methods; white-box, grey-box, and black-box. The white-box modeling is used as a common offline method. In this method, the synchronous generator is assumed to have a known structure with parameters which should be estimated. Many researches and standards have released the details of this method [5, 6]. In white-box modeling, offline tests such as short-circuit tests, standstill frequency response (SSFR) tests, standstill step-voltage test [7] and open circuit frequency response (OCFR) tests are applied to the synchronous machine [8]. These tests are usually difficult and time consuming. But the main drawback of the white-box modeling is that the parameters which are estimated in tests are not accurate enough for on-line applications.

In grey-box modeling, which included on-line measurements, a known structure is assumed for the synchronous machine, but the parameters are estimated using measured data [9-12]. An input signal is needed to apply it to machine. Like white-box modeling, grey-box modeling is encountered with changes in estimated parameters for different operating conditions.

To overcome the drawbacks of white- and grey-box modeling methods, the black-box modeling is used [13-16] in this paper. In the black-box modeling, the structure of the synchronous machine is unknown and the only concern is to map the input and output data regardless of machine physical concepts.

Different methods have been used in black-box modeling of a nonlinear system such as wavelets [17], Volterra series [18], genetic algorithm, neural networks [16, 19] and polynomials. In all aforementioned methods, input and output signals are sampled. A specific structure is chosen to constitute regressor matrix. The final goal is to estimate parameters corresponding to the regressors. Each method proposes a procedure to create regressor matrix and estimate the parameters. Matching pursuit is an auxiliary method added to estimation procedure in order to decrease the computation burden by means of estimating only the important parameters.

In this paper, a synchronous generator is modeled using black-box modeling method. Field voltage is the input signal and output signals are the active power and terminal voltage of the generator. A PMU is modeled on the generator bus which samples the voltage phasor signal with 5ms sampling rate. The polynomial structure is selected for modeling. The matching pursuit is used as an auxiliary method to reduce the computation burden while parameters are estimated using least square method.

This paper is organized as follows: Identification methods for Linear Time Invariant (LTI) and nonlinear systems and matching pursuit as an auxiliary method are discussed in the next section. The case study is described in Section 3. Simulation results are presented in Section 4 and the paper is concluded in the last section.

2. Identification Method

2.1. LTI Systems

For linear and time invariant systems, transfer function is defined according to the input and output signals as below:

$$G(s)=\frac{Y(s)}{X(s)}=\frac{b_0+b_1 s+\cdots+b_m s^m}{a_0+a_1 s+\cdots+a_n s^n} \quad (1)$$

where, for casual systems we have $n \geq m$. The difference equation is obtained from (1) as follows:

$$a_0 y_t+\cdots+a_n y_{t-n}=b_0 x_t+\cdots+b_m x_{t-m} \quad (2)$$

where, a_i and b_i are parameters which should be estimated in order to identify the system. To have unique solution, one of these parameters (here a_0) is assumed to be equal to 1.

By considering the effects of noise, approximations, etc. the equation is formed, as follows:

$$y_t=\underline{u}_t.\theta^T+e_t \quad (3)$$

where, t=1, 2, ..., N and N is the number of samples while:

$$\underline{u}_t=\begin{bmatrix}-y_{t-1} & \cdots & -y_{t-n} & x_t & \cdots & x_{t-m}\end{bmatrix} \quad (4)$$

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$$\theta = [a_0 \quad a_1 \quad \cdots \quad a_n \quad b_0 \quad b_1 \quad \cdots \quad b_m] \qquad (5)$$

and e_t shows the effects of noise, approximations, etc. $\underline{u}_t$ and θ are vectors in size of p.

To estimate $\hat{\theta}^T$, least square method is used as below:

$$\hat{\theta}^T = [\sum_{t=1}^{N} u_t . u_t^T]^{-1} \sum_{t=1}^{N} u_t . y_t \qquad (6)$$

2.2. Nonlinear Systems

For nonlinear systems such as synchronous generator the difference equation has a nonlinear structure. Different methods have been used such as wavelets [25], Volterra series, polynomials, neural networks, etc. In this paper, the second order polynomial is used to identify the study system. Therefore, the difference equation is formed using (3) as follows:

$$y_t = u_{1t}\theta_1 + u_{2t}\theta_2 + \cdots + u_{pt}\theta_p + u_{1t}^2\theta_{11} + \cdots + u_{pt}^2\theta_{pp} + u_{1t}u_{2t}\theta_{12} + \cdots + u_{p-1t}u_{pt}\theta_{p-1p} \qquad (7)$$

$$\underline{U}_t = [u_{1t} \quad \cdots \quad u_{pt} \quad u_{1t}^2 \quad \cdots \quad u_{pt}^2 \quad u_{1t}u_{2t} \quad \cdots \quad u_{p-1}u_p] \qquad (8)$$

$$\theta = [\theta_0 \quad \theta_1 \quad \cdots \quad \theta_p \quad \theta_{11} \quad \cdots \quad \theta_{pp} \quad \theta_{12} \quad \cdots \quad \theta_{p-1p}] \qquad (9)$$

$$y_t = \underline{U}_t . \theta^T + e_t \qquad (10)$$

and finally $\hat{\theta}^T$ is estimated with least square method using (6). In this study, we have n=4 and m=2. Therefore, the size of $\underline{u}_t$ is 7 (p=7). For aforementioned structure, the size of θ is 35. Thus, 35 parameters should be estimated.

2.3. Matching Pursuit Method

As discussed in previous section, to identify the system, parameters a_i and b_i should be estimated. Assume a LTI system which is represented by (2). Some of the coefficients (a_i and b_i) may be zero or near zero. Therefore, it is obvious that these coefficients do not play an important role in the behavior of the system. In addition, when n, m, and the order of polynomial (arbitrary structure) are increased, the number of parameters which should be estimated will increase numerously. Thus, it is very time consuming to estimate all parameters of the system.

For both linear and nonlinear systems, many methods have been proposed to identify the most important parameters before estimating all of them. These methods include Bootstrap method [20], F-test [21], matching pursuit method [22, 23] and ANOVA method [24].

Equation (7) has an expanded form for t=1, 2, . . ., N which is used in matching pursuit method as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} u_{11} & u_{21} & \cdots & u_{p-1,1}u_{p,1} \\ u_{12} & u_{22} & \cdots & u_{p-1,2}u_{p,2} \\ \vdots & \vdots & \vdots & \vdots \\ u_{1N} & u_{2N} & \cdots & u_{p-1,N}u_{p,N} \end{bmatrix} . \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_q \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} \qquad (11)$$

In our study, with n=4 and m=2 for second order polynomial structure, $\underline{U}_t$ has 35 columns (q=35) corresponding to 35 parameters and 35 regressors. These columns are represented with a vector named ϕ_i for i=1, 2, …, q. Thus there are 35 vectors in the space and each of them has a specific angle difference with $\underline{y}_t$. In matching pursuit method more important regressors are identified using these angles.

$$\phi_i = [U_{i,1} \quad U_{i,2} \quad \cdots \quad U_{i,N}]^T \qquad (12)$$

The flowchart shown in Fig. 1 has the following steps:

Step 1) A case study system with a specific operating condition is selected.

Step 2) Input and output signals are selected properly. In this study, the field voltage is the input signal and output signals are the output active power and terminal voltage. A pseudo-random binary sequence (PRBS) with a magnitude of 5% of the nominal field voltage is added to the input signal. The magnitude should be small to avoid any interference with normal operation of the system.

Step 3) The input signal is applied to the system and both input and output signals are sampled with 5ms sampling rate using a PMU.

Step 4) Select the structure and its details for the identification (second order polynomial with n=4 and m=2).

Step 5) Using N samples, constitute a relation like (11).

Step 6) Initialize k=1 and select M arbitrarily (M is the number of the most important regressors used for simulation. In this study, M=5).

Step 7) Compute angle between $\underline{\phi}_i$ and $\underline{y}_t$ as follows:

$$\cos\alpha_i = \frac{\underline{y}^T \cdot \underline{\phi}_i}{|\underline{y}| \cdot |\underline{\phi}_i|} \qquad i = 1,2,\cdots,q \qquad (13)$$

Step 8) The vector which has the minimum angle with the vector of output ($\underline{y}$) is selected and named $\underline{\phi}_k$. It can be proved that this regressor and its

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corresponding parameter (θ_k) have the most effect on the behavior of the system.

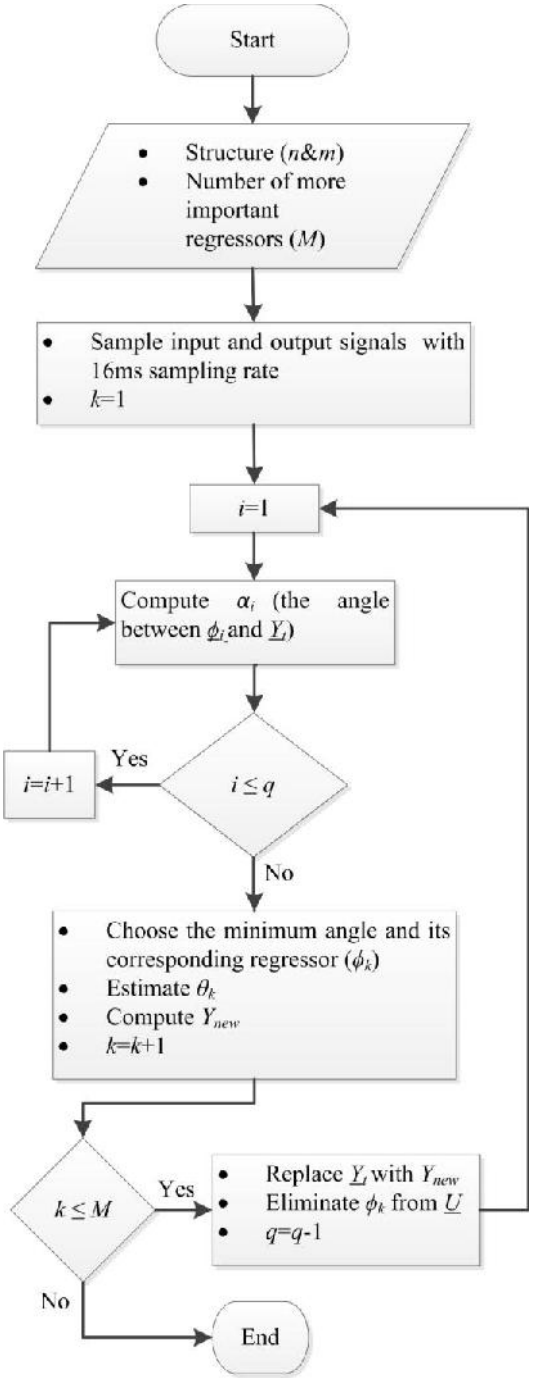


Fig. 1. Flowchart of proposed algorithm

Step 9) Assume that the system is modeled by only the most important regressor $\underline{\phi}_k$. Run the estimation using least square method to find $\hat{\theta}_k$ while:

$$\underline{y} = \theta_k \cdot \underline{\phi}_1 + \underline{e} \tag{14}$$

Step 10) Compute the error between measured output (y_t) and simulated output using $\hat{\theta}_k$ and name this error y_{new} as follows:

$$\underline{\hat{e}}_1 = \underline{y}_{new} = \underline{y} - \hat{\theta}_1 \cdot \underline{\phi}_1 \tag{15}$$

Step 11) Replace $\underline{y}$ with $\underline{y}_{new}$, eliminate $\underline{\phi}_k$ from regressors matrix ($\underline{U}$), q=q-1, k=k+1, and go to step 7 while k reaches M.

3. Case Study

As shown in Fig. 2, synchronous generator connected to an infinite bus and equipped by a PMU is studied in this paper. The machine has the following specifications: 30MVA, 13.8kV, 50Hz, and 2-pole. The PMU is considered to sample the phasors of voltage and current every 5ms.

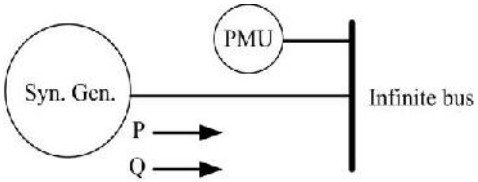


Fig. 2. Case study system

Following equations describe the model:

$$v_d = -r_a i_d + \dot{\varphi}_d - \omega\varphi_q \tag{16}$$

$$v_q = -r_a i_q + \dot{\varphi}_q + \omega\varphi_d \tag{17}$$

$$v_f = r_f i_f + \dot{\varphi}_f \tag{18}$$

$$0 = v_D = r_D i_D + \dot{\varphi}_D \tag{19}$$

$$0 = v_Q = r_Q i_Q + \dot{\varphi}_Q \tag{20}$$

where, v_d and v_q are direct-axis and quadratic-axis voltages, respectively, v_D and v_Q are dampers voltage, and v_f is the field voltage. φ_d and φ_q are direct-axis and quadratic-axis fluxes, φ_D and φ_Q are dampers fluxes, and φ_f is field flux. The dynamics of the rotor is as bellow:

$$\dot{\delta} = \omega \tag{21}$$

$$\dot{\omega} = \frac{1}{J}\left(T_m - T_e - D.\omega\right) \tag{22}$$

where, T_m is the mechanical input torque, T_e is the electrical output torque, J and D are inertia and damping factor, respectively, δ is the rotor angle, and ω is the rotor speed.

4. Simulation Results

The case study has been simulated by Simulink package of MATLAB. In order to apply the proposed method to the case study, a PRBS (pseudo-random binary sequence) signal with 5% of nominal value of field voltage is added to the field voltage. The field

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voltage (as the input signal) and the output active power and terminal voltage (as output signals) are sampled with the sampling rate of 5ms. These data are used for black-box modeling of the system. Measured input and output signals are shown in Figs. 3-5.

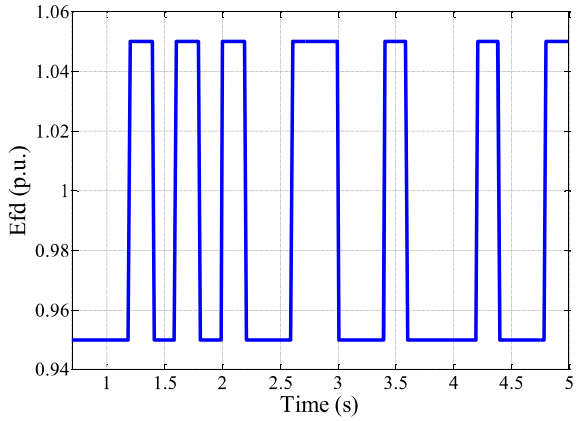


Fig. 3. Field voltage

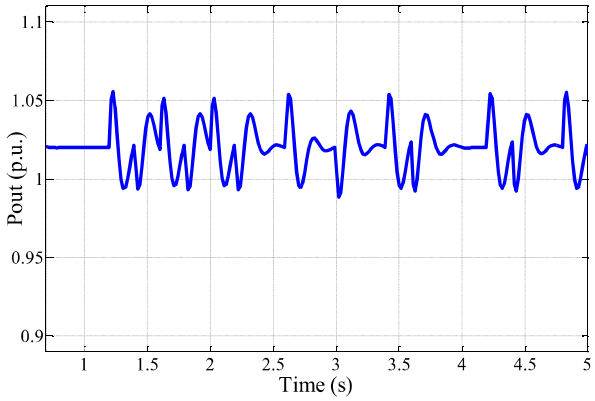


Fig. 4. Electrical power

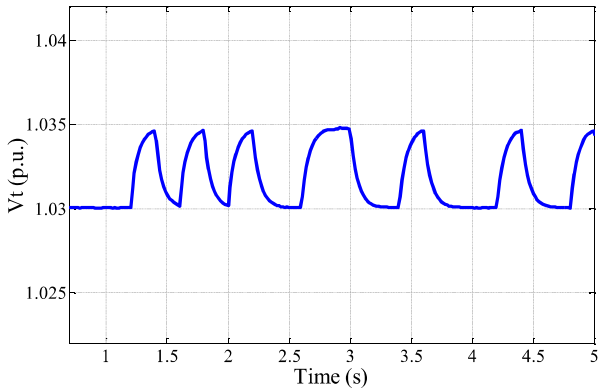


Fig. 5. Terminal voltage

A second degree polynomial structure with m=2 (numerator degree) and n=4 (denominator degree) is used to identify the system. Matching pursuit is used as an auxiliary method to reduce the computations. As mentioned before, with m=2 and n=4 for a second order polynomial, 35 parameters must be estimated to model the system. In this study, the 5 most important regressors and their 5 corresponding parameters are selected to be estimated. Parameter estimation is done using least square method. Simulation results for output active power and terminal voltage are presented in Fig. 6 and Fig. 7, respectively.

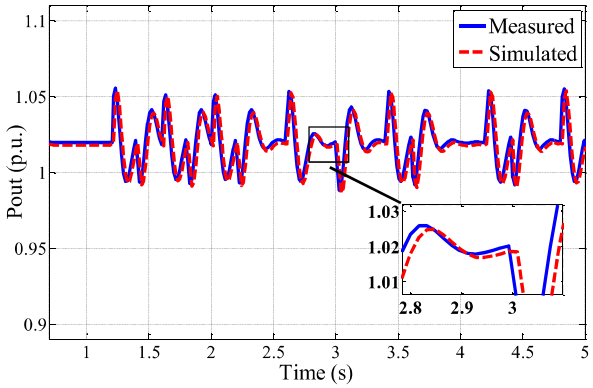


Fig. 6. Measured and simulated output active powers

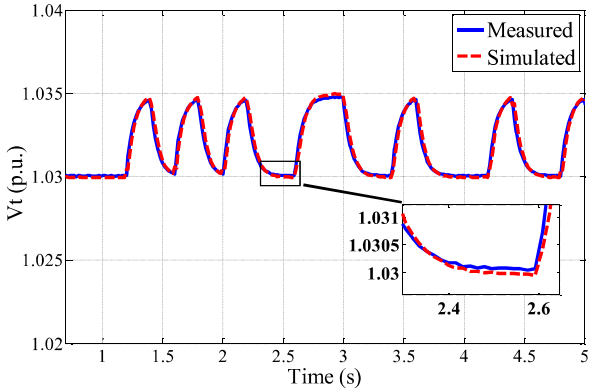


Fig. 7. Measured and simulated terminal voltages

The errors between measured and simulated values of the output active power and terminal voltage are presented in Fig. 8 and Fig. 9, respectively. The results show the high accuracy of the proposed method and confirm that this identification procedure can properly represent the system. Thus, the model will be useful for power system analysis and designing equipment such as power system stabilizer (PSS) and online predictive controllers.

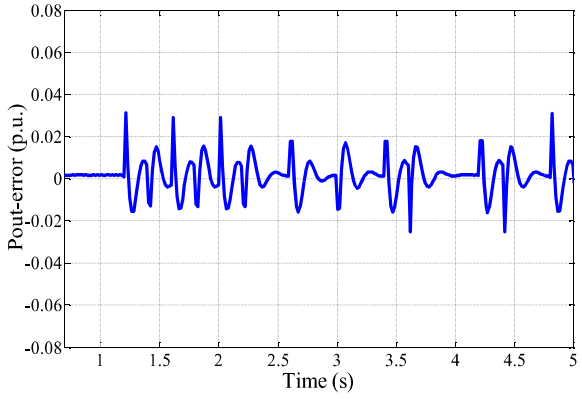


Fig. 8. Error signal of output power

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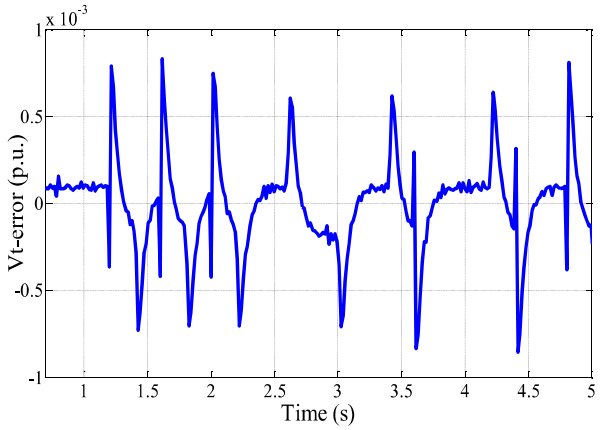


Fig. 9. Error signal of terminal voltage

To emphasize the accuracy of the proposed method, the minimum, and maximum, and average value of error signals are given in Table 1.

Table. 1. Values of error signals

Values (p.u.)	V_t	P_{out}
Maximum	0.000813	0.031
Minimum	-0.000882	-0.025
Average	-0.000032	0.0014

5. Conclusion

The application of the matching pursuit method for black-box modeling of a nonlinear synchronous generator has been proposed in this paper. A second degree polynomial structure has been used to represent the difference equation and consequently parameter estimation. The proposed algorithm has been applied to a nonlinear generator. The required signals were field voltage as the input signal and output active power and terminal voltage as output signals. A PMU has been used to record and sample the output signals with 5ms sampling rate. The results show the high accuracy and effectiveness of the obtained model. This model is useful for power system analysis and designing equipment such as power system stabilizer (PSS) and online predictive controllers.

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