

# Scheduling security constraint unit commitment for power system including stochastic wind power generation

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## Abstract :

This paper introduces a new approach for scheduling security constraint unit commitment (SCUC) including wind farms. Because of uncertainty in wind power production, we tried to develop a new method for incorporating wind power generation in power plant scheduling. For this, wind power generation modeled with unit commitment in a non-linear optimization problem and simulated by submitting different scenarios for wind farms. First, unit commitment solved in master problem. Then, scenarios for presenting volatile nature of wind power simulated. Numerical simulations show the effectiveness of supposed unit commitment for managing security of power system by considering volatility of wind power generation.

**Keywords:** security constraint unit commitment, wind farm, uncertainty in power system.

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## 1. Introduction

Today wind power became a popular energy in the world. At the end of year 2013, amount of power electric energy produced by wind power in the world was about 197 GW. Amount of wind power production is about 430 TW of annual that is 2.5% of electric energy in the world. In five past year, improving annual average in wind power was 27.6% that is been forecasted to year 2014, reach the amount of 3.35% and to 2018 reach 8% of whole electric power generation. Denmark with 21%, Portugal 18%, Spain 16%, Ireland 14% and Germany with 9% of wind power generation was stand at first places. In year 2013, 83 country used wind power for electric production. Being renewable energy, clean, cheap and so on is the reason that countries trended to take into consideration wind for electric power generation [1].

Beside the advantage of wind energy, uncertainty in electric power production is one of the concerns. Anyway, volatility of wind power may cause changes in power system characteristic such as voltage, frequency and current. It may also change the schedule of power plants and effect the system operation. These reasons make us to model the volatility and frequency of wind power [2-4]. There are different ways for forecasting wind power production that includes simulation, statistical method or combination of both. In simulation method, activity analysis used to forecast wind power. But in statistical method, artificial neural network or fuzzy logic are used, that need a large number of data sets or learning samples [5]. Wind power is predictable but not fully. Discussion on wind speed, prediction and analytical information, generator model and transmission network have many effects on operation of power system that is beyond of this paper. Management of wind farms and status of non-wind unit depend on behavior of wind and power produced by wind turbine. Obtained power from wind could be such that make a challenge in world power market.

Independent system operator (ISO) use different optimization method for manage the security of system with wind power [6-8]. Method of wind speed simulation use economical algorithm with security constraint to deal with wind power and other generator [9]. Suggested statistical method in [10] applied wind power with unit commitment and economic dispatch and did not consider transmission constraint. Statistical method in [11] for evaluating volatility of wind power introduced.

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## 2. Problem formulation

Proposed model for minimizing production cost

formulated with constraint in an optimization problem. Cost function including: cost of generators and startup and shutdown cost tried to be minimized with wind power and constraint in different operation hours. Large scale and non-linear nature of MINLP problem is the reason that bender decomposition technique is incorporated to the problem. Bender decomposition technique decompose problem into master problem and sub problem. First, master problem without considering constraint and wind power generation solved. Security constraint and wind power generation add to the problem in next step. Solution of problem according to the flowchart demonstrated as follow: solving master problem with Lagrange relaxation or mixed-integer programming or other method of optimization to schedule the power plants in base case. This answer would be checked in sub problem for different hours, if there was any violation a new constraint according to the new violation would be produce and impose to master problem. The master problem in new iteration would be solved with this new constraint. This process will be continuing until the optimized solution found. Fig. 1 show the submitted algorithm. In this algorithm the system reaction with dispatch of non-wind unit would be simulated.

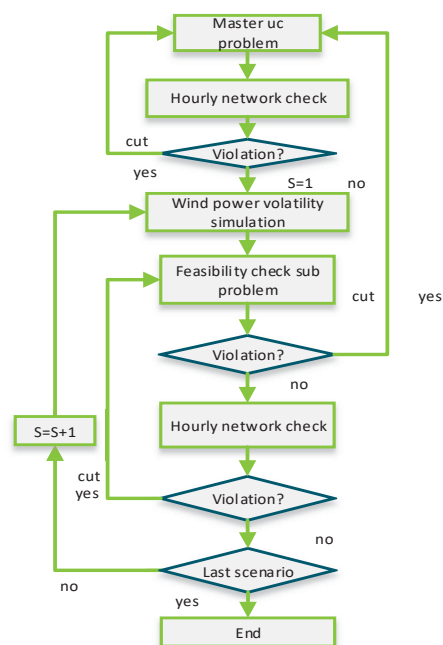


Fig. 1. main flowchart

### 2.1. Master problem

The objective of the master problem is to determine the day ahead schedule of generating units that minimize system production cost and satisfy system constraints. For modeling SCUC problem, objective function formed by combination of non-wind unit and wind power generation in an optimization problem structure with constraint as equation (1)-(16).

The objective functions (1) consist of fuel cost for electric power generation and startup and shutdown cost for each unit. This bracket included fuel cost, start-up and shut-down cost. The first term is the production cost  $F_{ci}(P_{it})$ , which will be calculated as the product of the heat rate (MBTU/h) and the unit's fuel cost (\$/MBTU). The second and third terms represents the start-up and shut-down cost for each unit which depends on the length of time that the unit is on or off. The start-up cost suppose zero for wind unit. Constraints of problem are as follow: power balance (2) this constraint would be imposes to insure that there is no power mismatch. System spinning and operating reserve requirement (3) and (4) that is defined as a fraction of system demands and a high operating limit of the largest on-line unit. Operating reserve capacity also includes interruptible loads. Unit ramping up and down limits (5) and (6). These constraints restrict the ramping rate of generation changes between any two successive hours. Unit minimum on and off time limits (7) and (8) indicate the minimum number of hours when the unit cannot be off/restored and constraint (9) specify unit generation limits [12]. Scenarios constraint would be listed as equations (10) to (14). Scenario power balance (10), scenario spinning and operating reserve (11), (12), permissible adjustment of real power generation (13) and constraint (14) restrict power generation in scenarios.

This is a mixed-integer, non-linear optimization problem. Incorporating wind power generation into SCUC and scenario increasing, make a large scale problem. So, bender decomposition technique imposes, to decrease the amount of computation. This technique decompose problem into master problem and sub problems.

$$\min \sum_{i=1}^{NG} \sum_{t=1}^{NT} [F_{ci}(P_{it}) * I_{it} + SU_{it} + SD_{it}] \quad (1)$$

Subject to:

$$\sum_{i=1}^{NG} P_{it} * I_{it} + \sum_{i=1}^{NW} P_{W,it}^f = \sum_{b=1}^{NB} D_{bt} + P_{L,t} \quad t = 1, 2, \dots, NT \quad (2)$$

$$\sum_{i=1}^{NG} R_{S,it} * I_{it} \geq R_{S,t} \quad t = 1, 2, \dots, NT \quad (3)$$

$$\sum_{i=1}^{NG} R_{o,it} * I_{it} \geq R_{o,t} \quad t = 1, 2, \dots, NT \quad (4)$$

$$P_{it} - P_{i(t-1)} \leq [1 - I_{i(t-1)}] UR_i + I_{i(t-1)} (1 - I_{i(t-1)}) P_{i,min} \quad (5)$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT \quad (6)$$

$$P_{i(t-1)} - P_{it} \leq [1 - I_{i(t-1)}] DR_i + I_{i(t-1)} (1 - I_{it}) P_{i,min} \quad (7)$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

$$[X_{i(t-1)}^{on} - T_i^{on}] [I_{i(t-1)} - I_{it}] \geq 0 \quad (8)$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

$$[X_{i(t-1)}^{off} - T_i^{off}] [I_{it} - I_{i(t-1)}] \geq 0 \quad (9)$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

$$P_{i,min} * I_{it} \leq P_{it} \leq P_{i,max} * I_{it} \quad (10)$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

$$\sum_{i=1}^{NG} P_{it}^s * I_{it} + \sum_{i=1}^{NW} P_{W,it}^s = \sum_{b=1}^{NB} D_{bt} + P_{L,t} \quad t = 1, 2, \dots, NT \quad (11)$$

$$\sum_{i=1}^{NG} R_{S,it} * I_{it} \geq R_{S,t} \quad t = 1, 2, \dots, NT \quad (12)$$

$$\sum_{i=1}^{NG} R_{o,it} * I_{it} \geq R_{o,t} \quad t = 1, 2, \dots, NT \quad (13)$$

$$|P_{it}^s - P_{it}| \leq \Delta_i \quad i = 1, 2, \dots, NG; t = 1, 2, \dots, NT \quad (14)$$

$$P_{i,min} * I_{it} \leq P_{it}^s \leq P_{i,max} * I_{it}$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

$$P_{i,min} * I_{it} \leq P_{it} \leq P_{i,max} * I_{it}$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

$$P_{i,min} * I_{it} \leq P_{it} \leq P_{i,max} * I_{it}$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

$$P_{i,min} * I_{it} \leq P_{it} \leq P_{i,max} * I_{it}$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

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$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

$$P_{i,min} * I_{it} \leq P_{it} \leq P_{i,max} * I_{it}$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

$$P_{i,min} * I_{it} \leq P_{it} \leq P_{i,max} * I_{it}$$

$$i = 1, 2, \dots, NG; t = 1, 2, \dots, NT$$

## 2.2. Sub problem

Solution of the master problem, i.e., the hourly unit commitment and dispatch, is used in the base case and contingencies. Network checks sub problems to examine the feasibility of the master problem solution for satisfying the network security and constraint. In the case of violations, hourly cuts are provided to the UC problem to ensure whether this solution accommodate with wind power or not. If any violation occurred, benders cut (15)-(17) generate to mitigate violation. Benders cut (16) check the security constraint whether it can be satisfied by status changing or not. Equation (17) generate to insure that wind power can satisfy constraint in unit commitment.

$$v^s(P) = v^s + \sum_{i=1}^{NG} \sum_{t=1}^{NT} \left( \frac{\partial v^s}{\partial P_{it}} | \hat{P}_{it} \right) (P_{it} - \hat{P}_{it}) \leq 0 \quad (15)$$

$$w(P, I) = W + \sum_{i=1}^{NG} \left( \frac{\partial W}{\partial P_{it}} | \hat{P}_{it} \right) (P_{it} - \hat{P}_{it}) \quad (16)$$

$$+ \sum_{i=1}^{NG} \left( \frac{\partial W}{\partial I_{it}} | \hat{I}_{it} \right) (I_{it} - \hat{I}_{it}) \leq 0$$

$$W^s(P) = W^s + \sum_{i=1}^{NG} \left( \frac{\partial W^s}{\partial P_{it}} | \hat{P}_{it} \right) (P_{it} - \hat{P}_{it}) \leq 0 \quad (17)$$

$$W^s(P) = W^s + \sum_{i=1}^{NG} \left( \frac{\partial W^s}{\partial P_{it}} | \hat{P}_{it} \right) (P_{it} - \hat{P}_{it}) \leq 0$$

$$W^s(P) = W^s + \sum_{i=1}^{NG} \left( \frac{\partial W^s}{\partial P_{it}} | \hat{P}_{it} \right) (P_{it} - \hat{P}_{it}) \leq 0$$

$$W^s(P) = W^s + \sum_{i=1}^{NG} \left( \frac{\partial W^s}{\partial P_{it}} | \hat{P}_{it} \right) (P_{it} - \hat{P}_{it}) \leq 0$$

$$W^s(P) = W^s + \sum_{i=1}^{NG} \left( \frac{\partial W^s}{\partial P_{it}} | \hat{P}_{it} \right) (P_{it} - \hat{P}_{it}) \leq 0$$

$$W^s(P) = W^s + \sum_{i=1}^{NG} \left( \frac{\partial W^s}{\partial P_{it}} | \hat{P}_{it} \right) (P_{it} - \hat{P}_{it}) \leq 0$$

$$W^s(P) = W^s + \sum_{i=1}^{NG} \left( \frac{\partial W^s}{\partial P_{it}} | \hat{P}_{it} \right) (P_{it} - \hat{P}_{it}) \leq 0$$

used to specify generation of each unit in every system operation hour [13-15].

### 3. Scenario generation

To simulate the volatility of wind nature, normal distribution  $N(\mu, \sigma^2)$  used and wind power subjected to. By using Monte Carlo different scenario easily generate and subject to normal distribution. Here  $\mu$  is variance, and  $\sigma^2$  standard deviation that shows the volatility of forecasted wind. This large number of generated scenarios included a wide computational area and may lead to a time consuming process. All of these generated scenarios are not feasible. So, by Latin hypercube sampling (LHS) technique before entering master problem, feasible solutions separated from the whole answer areas and computational time decrease by this reduction scenario with feasible distribution found [16-18].

Reducing scenarios will also reduce computational requirement for simulating [19].

### 4. Case studies

A modified IEEE 6-bus system with 3 generation unit and 7 branches and IEEE 24-bus system with 33 generation unit and 186 branches were analyzed to illustrate the proposed method.

#### 4.1. Six-bus system

**Case 1:** six-bus system in base case. Structure of this system without wind farm has been depicted in Fig. 2.

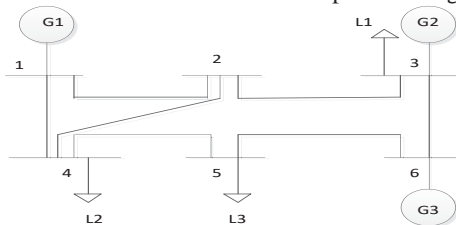


Fig. 2. six-bus system with wind farm

Generation unit are stand at buses 1, 3, 6 and buses number 2, 3, 4, 5 are load bus. Information of this network is shown in Table. 1, Table. 2, Table. 3 and Table. 4.

In base case, security constraint unit commitment solved without corporation of wind farms. Here, 20% of loads were assumed in bus 3, 10% in bus 2, 30% in bus 4 and the rest in bus 5. Total operational cost of 125381.327 \$ obtained. The problem converges in 3 min. Unit power generation, unit status and power distributions in different hours are shown respectively in Table 5, Fig. 3 and Fig. 4.

Table. 5. power generation

hour	T1	T2	T3	T4	T5	T6	T7	T8
P1	179	168	162	157	162	179	213	220
P2	0	0	0	0	0	0	0	0
P3	0	0	0	0	0	0	0	0
hour	T9	T10	T11	T12	T13	T14	T15	T16
P1	220	220	220	220	220	220	220	220
P2	0	0	10	12	10	12	12	10
P3	12/5	19/4	19/5	20	19/5	20	19/5	20
hour	T16	T17	T18	T19	T20	T21	T22	T23
P1	220	220	220	220	220	220	220	220
P2	10	10	10	0	0	0	0	0
P3	20	20	14/4	14/4	12/5	12/5	14/4	12/5

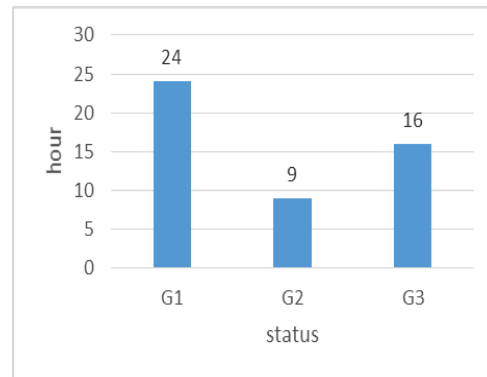


Fig. 3. unit status for 6-bus in base case

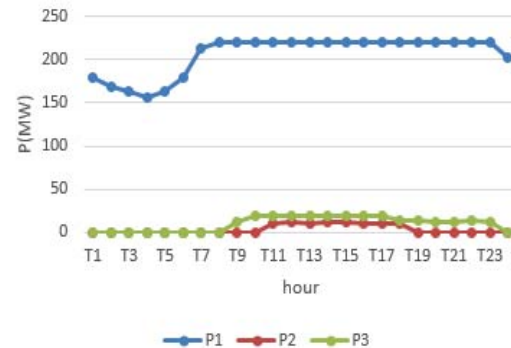


Fig. 4. power generation in base case

At the beginning of the day, demand is not so high and the first committed unit is only unit 1. By demand increasing maximum level of unit 1 produced in peak hours, and also unit 2 and 3 would turn on. In this case, 3 units with 49 operational hours satisfy loads.

**Case 2:** Six-bus system with wind farms. Structure of this system with wind farm has been depicted in Figure 5. Here, wind farm placed at bus 4.

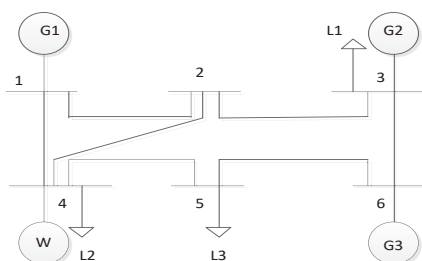


Fig. 5. six-bus system with wind farm

By using normal distribution and LHS, ten different scenarios produce have been shown in Table 6.

Table 6. wind power generation in different scenarios

hour	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
T1	44.3	47.9	50.3	47.6	39.0	42.0	48.3	41.6	46.6	44.7
T2	71.6	73.1	66.7	71.0	63.9	73.1	74.0	73.6	67.1	71.2
T3	75.5	73.8	77.1	75.5	67.7	73.6	79.7	73.5	79.7	74.5
T4	85.0	73.0	87.0	83.8	82.0	87.3	76.3	82.9	77.5	81.7
T5	82.8	81.2	81.6	91.0	86.0	87.2	85.4	79.0	86.4	75.9
T6	85.5	81.9	85.1	74.1	85.2	87.3	86.0	79.8	83.7	83.5
T7	101.4	105.7	97.8	96.7	92.1	101.9	98.1	105.5	101.3	105.3
T8	105.7	99.4	90.5	100.3	99.9	89.8	92.1	98.7	105.8	107.4
T9	80.1	85.1	82.9	77.7	74.0	78.5	73.3	73.2	77.0	85.0
T10	57.2	62.1	63.0	68.0	66.3	66.7	59.3	61.5	63.3	85.1
T11	98.8	101.2	102.7	107.4	95.5	99.3	102.7	96.3	87.9	105.4
T12	87.5	92.2	87.5	86.7	89.1	96.6	87.0	97.8	102.3	91.2
T13	90.3	85.8	90.5	85.3	85.1	79.2	78.0	85.0	78.3	79.1
T14	79.0	78.5	86.5	80.0	74.2	84.6	74.2	74.8	85.8	82.4
T15	73.5	81.8	80.3	75.7	84.1	80.2	78.0	72.8	79.1	79.8
T16	29.6	31.0	31.7	29.2	31.9	33.1	27.6	31.2	37.1	29.7
T17	3.7	4.3	4.5	3.7	4.1	4.3	4.3	4.0	3.7	4.1
T18	9.5	8.0	8.3	7.3	7.5	6.8	8.7	7.7	7.9	7.6
T19	11.4	9.8	11.3	10.0	9.5	9.7	8.8	11.8	10.1	9.3
T20	5.3	4.6	5.3	5.6	5.6	4.3	4.5	5.6	5.6	5.0
T21	6.4	5.6	5.7	5.7	7.0	6.3	6.8	5.6	6.3	6.1
T22	58.4	50.7	52.0	52.1	58.6	57.6	57.2	54.2	57.9	55.7
T23	79.6	74.7	87.8	81.8	80.3	78.3	83.4	88.4	83.3	90.3
T24	55.1	49.7	48.1	52.9	51.9	49.7	54.5	56.5	56.7	53.3

Two scenarios choose for results discussion. After solving problem, operational cost for first scenario, obtained 117124.403 \$ and 7<sup>th</sup> scenario 108321.147 \$ that show a reduction in comparison to the base case. Unit 3 turned off and operational hours reduced to 48 hour for first scenario and 27 for 7<sup>th</sup> scenario. Difference in costs, unit production level or unit status related to different amount of wind penetration by each scenarios. The results changes and unit power generation for two scenario and unit status in different scenarios respectively depicted in Table 7 and Fig. 6.

Table 7. power generation in two scenario

hour	S1			s7		
	P11	P21	P31	P17	P27	P37
T1	119/2	10	0	116/6	10	0
T2	110	10	0	108/5	10	0
T3	105/4	10	0	107	10	0
T4	100/8	10	0	96	0	0
T5	100/8	10	0	98/7	0	0
T6	106/4	10	0	103/2	0	0
T7	123/2	10	0	112/6	0	0
T8	156/8	10	0	148/1	0	0
T9	187/6	10	0	176/5	0	0
T10	210/5	10	0	195/2	0	0
T11	206/7	10	0	196/4	0	0
T12	209	10	0	203	0	0
T13	207/7	10	0	205	0	0
T14	209	10	0	207	0	0
T15	209	10	0	207	0	0
T16	200/6	10	0	198/6	0	0
T17	197/8	10	0	194/5	0	0
T18	197/8	10	0	193/2	0	0
T19	189/4	10	0	181/2	0	0
T20	188/6	10	0	179/3	0	0
T21	188/6	10	0	178/5	0	0
T22	189/4	10	0	175	0	0
T23	177/6	10	0	172	0	0
T24	145/6	10	0	145/6	0	0

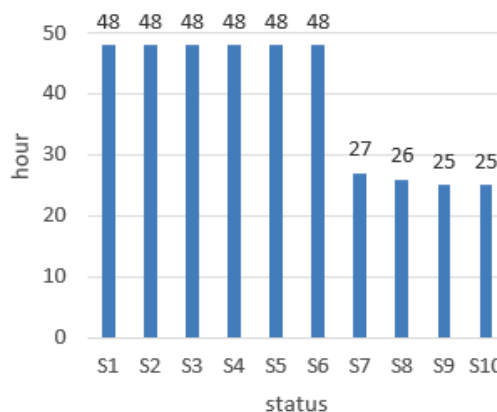


Fig. 6. unit status in different scenarios

Because of wind penetration, operational hours differ from one scenario to another one. The weighted coefficient of permeability in scenarios S7 to S10 is the cause of time reduction. With increasing wind permeability, more loads will satisfy and power plants with high marginal cost will turned off. Operational hours in some scenarios are near the base case, but marginal cost is less than base case. Because start up and shut down cost decrease and some unit produce power at the lowest range. Also, power distribution in different hours for two scenarios is shown in Fig 7 and Fig. 8 that illustrate this power reduction.



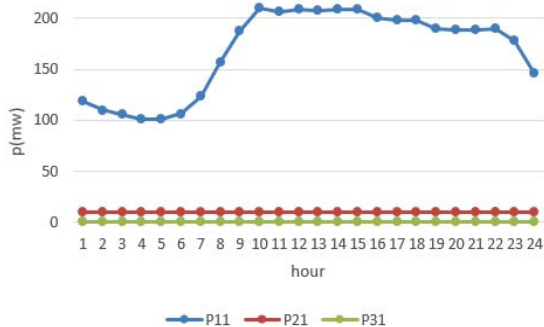


Fig 7: power generation in first scenario

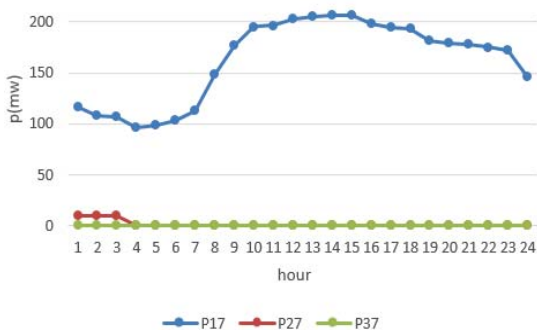


Fig. 8. power generation in 7<sup>th</sup> scenario

## 4.2. 24-bus system

**Case 1:** 24-bus system in base case.

Total operational cost of 373869.23 \$ with 742 operational hours and 18 min of processing time obtained. Just 32 units participate in power generation. Unit 15 is off all 24 hour. Unit 4 for 15 hour, Unit 21 for 18 hour, Units 18 and 19 for 19 hour and other Units are on for the whole time. Unit status is shown in Fig. 9.

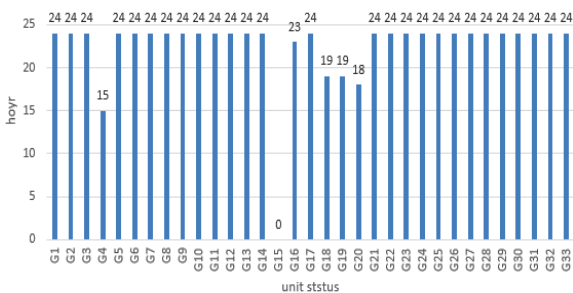


Fig. 9. unit status for 24-bus system in base case

**Case 2:** 24-bus system with wind farm.

Like previous case, by using normal distribution and LHS and statistical method, ten different scenario produce for incorporating to SCUC. For analysis and discussion five scenarios picked up and compared with base case. For first scenario, total operational cost of 355567.937 \$ with 663 operational hours and 31 min

of processing time obtained. In comparison to the base case has a reduction of 18301.35 \$ in cost and 79 hours in operational hour. Just 31 units participate in power generation. Power schedule change and, Unit 15 and 14 is off all 24 hour, Unit 11 for 12 hour, Unit 13 for 8 hour, Units 23 for 8 hour, Units 24 for 12 hour, Unit 31 for 13 hour, Unit 33 for 10 hour and other Units are on for the whole time. Unit status is shown in Figure 10.

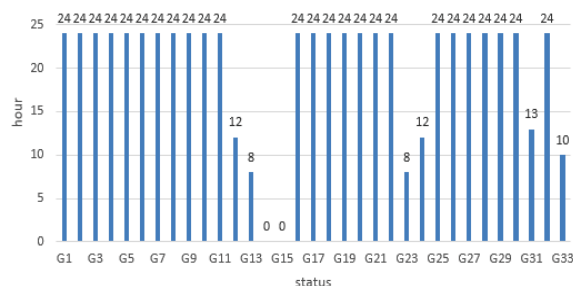


Fig. 10. unit status for 24-bus system with wind power

A total comparison with reduction in cost and consuming time for two cases is shown in Table 8.

Table 8: comparison of two cases

6-bus system	operational cost	cost reduction	time
base case	125381.327 \$	0\$	3 min
first scenario	124421.209\$	960.118\$	7 min
3th scenario	123469.114\$	1912.213\$	7 min
7th scenario	108321.147\$	17060.17\$	6 min
8th scenario	108123.568\$	1725.759\$	6 min
9th scenario	107654.243\$	17727.084\$	6 min
24-bus system	operational cost	cost reduction	time
base case	373869.230 \$	0\$	18 min
first scenario	355567.937 \$	18301.293\$	31 min
3th scenario	367733.356\$	6135.874\$	32 min
7th scenario	355467.741\$	18401.489\$	31 min
8th scenario	355166.678\$	18702.552\$	31 min
9th scenario	346521.254\$	27347.976\$	29 min

## 5. Observation

- Decrease in total operational cost.
- Total operational hour decrease and few on/off unit status is required.
- Reduction in fuel consumption due to the lower operational hour.
- Lower system emission limit.
- Lower congestion in transmission line.

## 6. Conclusion

In this paper, stochastic wind power generation incorporated into SCUC algorithm. Results show that this ability could reduce system operational cost and fuel consumption. Also, congested transmission line decrease and need few on/off generating unit.

Formulation of SCUC consist of objective function and constraint with benders decomposition technique. Iteration process between master problem and sub problems could lead to an optimized generation unit status with minimum cost. The processing time by adding wind power increase, because the amount of computation increase. But this time for problem convergence is still good and is about 3 min for 6 bus and about 18 min for 24 bus system.

## 7. Nomenclature

$i$	index for unit.
$t$	index for buses.
$s$	superscript for scenario.
NG	number of unit.
NT	number of period (hour).
$DR_i$	Ramp-down rate limit of unit $i$ .
$UR_i$	Ramp-up rate limit of unit $i$ .
$F_{ci}(0)$	Production cost function for unit $i$ .
$I_{it}$	Commitment state of unit $i$ at time $t$ .
$P_{D,t}$	System demand at time $t$ .
$P_{W,it}^f$	Forecasted wind power of unit $i$ at time $t$ .
$P_{W,it}^s$	Simulated wind power of unit $i$ at time $t$ in scenario $s$ .
$P_{L,t}$	System losses at time $t$ .
$P_{it}$	Power production of unit $i$ at time $t$ .
$P_{i,max}$	Lower real power generation of unit $i$ .
$P_{i,min}$	Upper real power generation limit of unit $i$ .
$R_{S,t}$	System spinning reserve at time $t$ .
$R_{O,t}$	System operating reserve at time $t$ .
$R_{S,it}$	Spinning reserve of unit $i$ at time $t$ .
$R_{O,it}$	Spinning reserve of unit $i$ at time $t$ .
$SU_{it}$	Startup cost of unit $i$ at time $t$ .
$SD_{it}$	Shutdown cost of unit $i$ at time $t$ .
$T_i^{off}$	Minimum off time of unit $i$ .
$T_i^{on}$	Minimum on time of unit $i$ .
$X_{it}^{off}$	Off time of unit $i$ at time $t$ .
$X_{it}^{on}$	On time of unit $i$ at time $t$ .
$\Delta_i$	Permissible real power adjustment of unit $i$ .
$w, v$	Power mismatch.
$\wedge$	Given variable.

## Appendix

Table. 1. generator data

Unit	Bus.no	pmin(mw)	pmax(mw)	Qmin(mw)	Qmax(mw)	Ramp
G1	1	220	90	80-	200	100
G2	2	150	10	40-	70	50
G3	6	60	5	40-	50	50

Table. 2. generator data

Unit	C	B	A	Mon off	Min on	Ini.state	Start up cost
G1	41/22	82/2	49	4	4	1	124
G2	85/16	97/8	12	2	3	1	345
G3	20/17	85/6	24	1	1	1	0

Table. 3. load data

Hr	Pd(MW)	Hr	Pd(MW)
1	219	13	327
2	234	14	324
3	234	15	327
4	237	16	288
5	240	17	261
6	243	18	246
7	273	19	255
8	291	20	237
9	285	21	243
10	282	22	282
11	330	23	282
12	327	24	249

Table. 4. line data

Line no.	From bus	To bus	R(p.u)	X(p.u)	Flow limit(Mw)
1	1	2	5	17	200
2	1	4	3	258	100
3	2	4	7	197	100
4	5	6	2	14	100
5	2	3	0	37	100
6	4	5	0	37	100
7	3	6	0	18	100

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