Decentralized Adaptive Control of Large-Scale Non-affine Nonlinear Time-Delay Systems Using Neural Networks

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Abstract:

In this paper, a decentralized adaptive neural controller is proposed for a class of large-scale nonlinear systems with unknown nonlinear, non-affine subsystems and unknown nonlinear time-delay interconnections. The stability of the closed loop system is guaranteed through Lyapunov-Krasovskii stability analysis. Simulation results are provided to show the effectiveness of the proposed approaches.

Keywords: Adaptive decentralized control, neural networks, non-affine nonlinear large-scale systems, time-delay systems.

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1. Introduction

In the recent years, there has been an increasing interest in developing theory of decentralized control for large-scale systems. Decentralized control issues naturally arise from controlling many complex systems found in the power industry, aerospace and chemical engineering applications, telecommunication network, and so on. The main advantage of decentralized control is that they can alleviate the computational burden associated with a centralized control and enhance the robustness and reliability against interacting operation failures [1]. Knowing that most of actual large-scale systems are nonlinearly coupled to the dynamics of the processes, researchers are still trying to control these systems [2-6]. Most of them either investigated subsystems that are linear in a set of unknown parameters [2-4], or considered isolated subsystems to be known [5, 6].

Based on the fact that nonlinear functions and the nonlinear interconnections parameters of subsystems in a large-scale system are almost unknown, thus in the literature neural networks (NNs) and fuzzy models have been considered as general tools for modeling nonlinear functions [7-9]. In [10], an indirect adaptive control method using self-recurrent wavelet NNs has been proposed for nonlinear dynamic systems. In [11], an adaptive single neural controller has been presented for a class of uncertain nonlinear systems subject to a nonlinear input. For multi input-multiple output (MIMO) non-affine nonlinear systems with completely unknown dynamics, an adaptive fuzzy control approach for was proposed in [12]. The Authors in [13] have considered a neuro-fuzzy network with dynamical structure to solve the adaptive tracking problems of MIMO uncertain nonlinear systems. In [14], radial basis NNs were utilized for a class of nonlinear decentralized large-scale systems with unknown subsystems. Also, the authors in [15] proposed a decentralized neuro-adaptive control scheme for large-scale non-affine nonlinear systems with unknown dynamics under the assumption that the interconnections are unknown high-order, nonlinear functions.

Beside the uncertainty and nonlinearity, time delay is an impressive issue in many physical and technological systems, particularly in large-scale systems. Regarding the information transmission among subsystems, time-delay often causes deterioration of control system performance. Moreover, time delay is one major potential source of instability in practical system. Therefore, some studies have focused on decentralized control of time-delayed large-scale systems, such as [16-19]. In [20], a dynamic output feedback tracking control problem was studied for stochastic interconnected time-delay systems. In [21], a decentralized adaptive output feedback control scheme was proposed for a class of interconnected nonlinear time delay systems with subsystems involving unknown parameters and being preceded with hysteresis described by the saturated PI model. In [22], the tracking control of a class of time delay large-scale systems with output-feedback by utilizing backstepping technique has been investigated. In [23, 24], the problem of model reference control of large-scale systems with time delays was considered, while the isolated subsystems of the considered large-scale systems were linear with stringent matching conditions.

This paper proposes a decentralized neuro-adaptive control schemes for large-scale non-affine nonlinear systems with unknown time-varying delay interconnections. The NNs are used to compensate the unknown nonlinear interactions. This paper also carries out a stability analysis of the closed-loop system based on Lyapunov-Krasovskii stability theory to make sure that proposed decentralized adaptive control scheme makes all the signals in the closed-loop system bounded and tracking errors asymptotically tend to zero.

The rest of the paper is organized as follows: problem formulation and derivation of the error dynamics are introduced in Section 2. In Section 3, the main results of decentralized adaptive neural network control and stability analysis are presented. Simulation results are given in Section 4. Section 5 concludes the paper.

2. Problem Formulation and Derivation of the Error Dynamics

Consider a large-scale nonlinear system composed of N interconnected subsystems described by

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \vdots \\ \dot{x}_{i,n_i} = f_i(\mathbf{x}_i, u_i) + \sum_{j=1}^{N} h_{i,j}(\mathbf{x}_j(t-\tau_{i,j}(t))) \end{cases}$$
$$y_i = x_{i,1} \tag{1}$$

where $\mathbf{x}_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}]^T$, $1 \le i \le N$, is the state vector of the ith subsystem; $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the input and output signals, respectively. The function $f_i(\mathbf{x}_i, u_i)$ is unknown and sufficiently smooth and $\sum_{j=1}^{N} h_{i,j}(\mathbf{x}_j(t-\tau_{i,j}(t)))$ denotes unknown nonlinear time-delay interconnection among subsystems where $\tau_{i,j}(t)$ is time-varying delay satisfying $\tau_{i,j}(t) \le \tau$, $\dot{\tau}_{i,j}(t) \le \tau_{\max} < 1$ with τ and $\tau_{\max}$ are known constants.

Assumption 1: The desired continuous time trajectory vector $x_{i,1}^{d}$ and its time-derivatives up to order $n_i - 1$ for $i = 1, \ldots, N$, are given and bounded.

Assumption 2: For each subsystem, there exist a positive constant f_i^L such that [25]:

$$0 < f_i^L \le \frac{\partial f_i(\mathbf{x}_i, u_i)}{\partial u_i} \tag{2}$$

and H_i such that

$$\left| \frac{d}{dt}\left[\frac{\partial f_i(\mathbf{x}_i, u_i)}{\partial u_i} \right] \right| \le H_i < \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)} f_i^L, \quad \forall (\mathbf{x}_i, u_i) \in \Pi \times \mathbb{R}, \Pi \subset \mathbb{R}^{n_i} \tag{3}$$

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where Q_i and P_i are the positive-definite matrices properly selected by the user. Equation (2) is a direct extension of Assumption 1 in [26] for affine systems, Assumption 2 for an affine system implies that the input gain must be bounded and nonsingular. Note that this condition is also obviously valid for an LTI system. A condition on the rate of change is also used in Assumption 1 of [26]. Although condition [22] seems to be restricting, it is more applicable to the special class of non-affine systems rather than affine systems in this paper.

Assumption 3: The interconnection $\sum_{j=1}^{N} h_{i,j}(\mathbf{x}_j(t-\tau_{i,j}(t)))$ is bounded by:

$$\left|h_{i,j}(\mathbf{x}_j(t-\tau_{i,j}(t)))\right| \le \eta_{i,j}(\mathbf{e}_j^T(t-\tau_{i,j}(t))P_j\mathbf{b}_j) \quad (4)$$

where $\eta_{i,j}(\mathbf{e}_j^T P_j \mathbf{b}_j)$, $i=1,...,N$, $j=1,...,N$ is an unknown smooth nonlinear function. This assumption indicates that the interaction term in the *i*th subsystem must be bounded by some arbitrary functions of a certain form of variables. This form can be expressed in the terms of a linear combination of errors generated by other subsystems.

Let $\sum_{j=1}^{N} h_{i,j}(\mathbf{x}_j(t-\tau_{i,j}(t))) = 0$ in (1), and then the isolated subsystem would be obtained as follows:

$$\dot{x}_{i,n_i} = v_i, \qquad v_i = f_i(\mathbf{x}_i, u_i^*) \quad (5)$$

where u_i^* is the ideal control function and v_i is commonly referred to as the pseudo control signal. The pseudo control system is chosen in this derivation as a linear operator. Generally, It may be nonlinear, for example if a sliding mode component is included [27, 28]. The transformation (5) is defined locally by invoking the implicit function theorem [29]. Since, the pseudo control signal v_i is not generally a function of the control signal u_i but rather a state dependent operator, and reminding assumption 2 one would have:

$$\frac{\partial\left[v_i - f_i(\mathbf{x}_i, u_i^*)\right]}{\partial u_i} \neq 0 \quad (6)$$

The fact that the expression in (6) is nonsingular implies that in neighborhood of every $(\mathbf{x}_i, u_i) \in \Pi \times \mathbb{R}$, there exists an implicit function $\alpha(\mathbf{x}_i, v_i)$ such that:

$$v_i - f_i(\mathbf{x}_i, \alpha(\mathbf{x}_i, u_i)) = 0 \quad (7)$$

and

$$u_i^* = \alpha(\mathbf{x}_i, v_i) \quad (8)$$

The union of all such neighborhood can be utilized to extend the existence of the transformation to the entire domain. Let the tracking error be $e_i = y_{i,d} - y_i$, where $y_{i,d}$ is the desired output and vectors $\mathbf{e}_i, \dot{\mathbf{e}}_i$ are defined as

$$\mathbf{e}_i = [e_i, \dot{e}_i, ..., e_i^{(n_i-1)}]^T \quad (9)$$

$$\dot{\mathbf{e}}_i = [\dot{e}_i, e_i^{(2)}, ..., e_i^{(n_i)}] \quad (10)$$

From tracking error definition one would have:

$$\begin{aligned} e_i^{(n_i)} &= y_i^{(n_i)} - y_{i,d}^{(n_i)} = \dot{x}_{i,n_i} - y_{i,d}^{(n_i)} \\ &= f_i(\mathbf{x}_i, u_i) + \sum_{i=1}^{N} h_{i,j} - y_{i,d}^{(n_i)} \end{aligned} \quad (11)$$

From Mean Value Theorem in [30], it is obvious that there exists $\lambda_i \in (0,1)$ such that

$$f_i(\mathbf{x}_i, u_i) = f_i(\mathbf{x}_i, u_i^*) + (u_i - u_i^*) f_{u_i} \quad (12)$$

where $f_{u_i} = \left[\partial f_i(\mathbf{x}_i, u_i)/\partial u_i\right]\Big|_{u_i = u_{\lambda_i}}$ with

$u_{\lambda_i} = \lambda_i u_i + (1-\lambda_i) u_i^*$.

Substituting (12) into (11), one obtains:

$$e_i^{(n_i)} = f_i(\mathbf{x}_i, u_i^*) + (u_i - u_i^*) f_{u_i} + \sum_{i=1}^{N} h_{i,j} - y_{i,d}^{(n_i)} \quad (13)$$

Substituting (5) into (13), it can conclude that:

$$e_i^{(n_i)} = v_i + (u_i - u_i^*) f_{u_i} + \sum_{i=1}^{N} h_{i,j} - y_{i,d}^{(n_i)} \quad (14)$$

The pseudo control v_i is design as

$$v_i = -(a_{i,0} e_i + a_{i,1}\dot{e}_i + \cdots + a_{i,n_i-1} e_i^{(n_i-1)}) + y_{i,d}^{(n_i)} \quad (15)$$

where the coefficients are chosen, so that each $L_i(s) = s^{n_i} + a_{i,n_i-1}s^{n_i-1} + a_{i.n_i-2}s^{n_i-2} + \cdots + a_{i,0}$ has its roots in the open left-half plane, i.e. L_i is Hurwitz. From (14) and (15), one has:

$$\begin{aligned} e_i^{(n_i)} = &-(a_{i,0} e_i + a_{i,1}\dot{e}_i + \cdots + a_{i,n_i-1} e_i^{(n_i-1)}) \\ &+ (u_i - u_i^*) f_{u_i} + \sum_{i=1}^{N} h_{i,j} \end{aligned} \quad (16)$$

Substituting (16) into (10) the error dynamics can be further written in a matrix form as follows:

$$\dot{\mathbf{e}}_i = A_i \mathbf{e}_i + \mathbf{b}_i \left[(u_i - u_i^*) f_{u_i} + \sum_{i=1}^{N} h_{i,j} \right] \quad (17)$$

where A_i is Hurwitz matrix with the following form:

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_{i,0} & -a_{i,1} & -a_{i,2} & \cdots & -a_{i,n_i-1} \end{bmatrix} \quad (18)$$

and $\mathbf{b}_i = [0 \ 0 \ \cdots \ 1]^T$. Since A_i is Hurwitz, a unique positive-definite solution P_i to the following Lyapunov equation exists:

$$A_i^T P_i + P_i A_i = -Q_i \quad (19)$$

where the matrix Q_i is an arbitrary positive-definite matrix.

3. Decentralized Adaptive Neural Networks Design and Stability Analysis

This section presents a NN-based controller for (17) with unknown time-delay interconnection functions. Note that

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f_i and f_{u_i} are also assumed to be unknown. The ideal local control signal u_i^* may be represented by a Radial Basis Function Neural Networks (RBNN) or any approximation structure such that

$$u_i^* = \mathbf{B}_i^{*T} \Psi_{b_i}(\mathbf{z}_i) + u_{i,k}(\mathbf{x}_i) + \varepsilon_i$$
$$\mathbf{z}_i = [\mathbf{x}_i^T, v_i]^T \tag{20}$$

where $\Psi_{b_i}(\mathbf{z}_i) = [\psi_{b_{i,1}}(\mathbf{z}_i), \psi_{b_{i,2}}(\mathbf{z}_i), ..., \psi_{b_{i,k_i}}(\mathbf{z}_i)]^T \in \mathbb{R}^{K_i}$, is the neural network (NN) basis vector, and $\mathbf{B}_i^{*T}$ is the vector of ideal control parameters [26]. The term $u_{i,k}(\mathbf{x}_i)$ is a prior control term developed based on a prior model (experience) to improve the initial control performance.The integer K_i denotes the NN's number of nodes, and the term ε_i is called the NN approximation error satisfying $|\varepsilon_i| \le \varepsilon_{M_i}$, $\varepsilon_{M_i} > 0$. Then, an adaptive algorithm is proposed as

$$u_i = \hat{\mathbf{B}}_i^T \Psi_{b_i}(\mathbf{z}_i) - \mathrm{sgn}(\mathbf{e}_i^T P_i \mathbf{b}_i)\hat{\mathbf{C}}_i^T \Psi_{c_i}(\mathbf{e}_i^T P_i \mathbf{b}_i) + u_{i,k}(\mathbf{x}_i) - \frac{N}{2\left(f_i^L\right)^2}\mathbf{e}_i^T P_i \mathbf{b}_i + u_{i,R} \tag{21}$$

with

$$u_{i,R} = -\hat{\zeta}_i \,\mathrm{sgn}(\mathbf{e}_i^T P_i \mathbf{b}_i) - \hat{\theta}_i \,\mathrm{sgn}(\mathbf{e}_i^T P_i \mathbf{b}_i) \tag{22}$$

In (21), $\hat{\mathbf{B}}_i^T \Psi_{b_i}(\mathbf{z}_i)$ represents a RBNN employed to approximate the ideal controller for system and the $(\mathbf{e}_i^T P_i \mathbf{b}_i)\hat{\mathbf{C}}_i^T \Psi_{c_i}(\mathbf{e}_i^T P_i \mathbf{b}_i)$ term is used to compensate for the interconnection nonlinearity. The term $u_{i,k}(\mathbf{x}_i)$ is a prior continuous controller (possibly Proportional-Integral (PI), Proportional-Integral Derivative, or some other type of controllers) designed in advanced via heuristics or past experiences with the application of conventional control methods, and $N\mathbf{e}_i^T P_i \mathbf{b}_i / 2(f_i^L)^2$, $\hat{\zeta}_i \,\mathrm{sgn}(\mathbf{e}_i^T P_i \mathbf{b}_i)$ and $\hat{\theta}_i \,\mathrm{sgn}(\mathbf{e}_i^T P_i \mathbf{b}_i)$ are utilized for countering uncertainties in the NN approximation error and system interconnections. The following adaptive rules are proposed to update the parameters $\hat{\mathbf{B}}_i^T$, $\hat{\mathbf{C}}_i^T$, $\hat{\zeta}_i$ and $\hat{\theta}_i$:

$$\dot{\hat{\mathbf{B}}}_i = -\Gamma_{b_i} \mathbf{e}_i^T P_i \mathbf{b}_i \Psi_{b_i}(\mathbf{z}_i) \tag{23}$$

$$\dot{\hat{\mathbf{C}}}_i^T = \Gamma_{c_i} \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \Psi_{c_i}(\mathbf{e}_i^T P_i \mathbf{b}_i) \tag{24}$$

$$\dot{\hat{\zeta}}_i = \gamma_{\zeta_i} \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \tag{25}$$

$$\dot{\hat{\theta}}_i = \gamma_{\theta_i} \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \tag{26}$$

where $\Gamma_{b_i} = \Gamma_{b_i}^T > 0$, $\Gamma_{c_i} = \Gamma_{c_i}^T > 0$, $\gamma_{\zeta_i} > 0$ and $\gamma_{\theta_i} > 0$ are constant design parameters.

Lemma 1: Consider system (1) satisfying the conditions given in Assumption 2. The following inequality holds for each subsystem:

$$\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} + \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2{f_{u_i}}^2} > 0 \tag{27}$$

Proof: see [25].

Theorem: Consider a decentralized system comprising *N* subsystems described by (1) for which Assumptions 1–3 hold. Then, the control law (21) with adaptation laws (23)-(26) makes the tracking error asymptotically converge to zero and all signals in the closed-loop system are bounded.

Proof: Consider the following Lyapunov-Krasovskii function

$$V = \sum_{i=1}^{N}(V_{i,1} + V_{i,2} + V_{i,3}) \tag{28}$$

with

$$V_{i,1} = \frac{\mathbf{e}_i^T P_i \mathbf{e}_i}{2f_{u_i}}, \tag{29}$$

$$V_{i,2} = \frac{1}{2(1-\tau_{\max})}\sum_{j=1}^{N}\int_{t-\tau_{i,j}(t)}^{t} \eta_{i,j}^2 (\mathbf{e}_j^T(\tau) P_j \mathbf{b}_j) d\tau \tag{30}$$

$$V_{i,3} = \frac{1}{2}\left[\tilde{\mathbf{B}}_i^T \Gamma_{b_i}^{-1} \tilde{\mathbf{B}}_i + \tilde{\mathbf{C}}_i^T \Gamma_{c_i}^{-1} \tilde{\mathbf{C}}_i + \frac{\tilde{\zeta}_i^2}{\gamma_{\zeta_i}} + \frac{\tilde{\theta}_i^2}{\gamma_{\theta_i}}\right] \tag{31}$$

where $\tilde{\mathbf{B}}_i = \hat{\mathbf{B}}_i - \mathbf{B}_i^*$, $\tilde{\mathbf{C}}_i = \hat{\mathbf{C}}_i - \mathbf{C}_i^*$, $\tilde{\theta}_i = \hat{\theta}_i - \delta_{M_i}$ and $\tilde{\zeta}_i = \hat{\zeta}_i - \varepsilon_{M_i}$, (δ_{M_i} will be explained later.) and use the error dynamic (17) to write the time derivative of *V* as

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N}\left[\frac{1}{2f_{u_i}}\left(\dot{\mathbf{e}}_i^T P_i \mathbf{e}_i + \mathbf{e}_i^T P_i \dot{\mathbf{e}}_i\right) - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + \dot{V}_{i,2} + \dot{V}_{i,3}\right] \\ &= \sum_{i=1}^{N}\left[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + \mathbf{e}_i^T P_i \mathbf{b}_i (u_i - u_i^*) + \frac{\mathbf{e}_i^T P_i \mathbf{b}_i \sum_{j=1}^{N} h_{i,j}}{f_{u_i}} + \dot{V}_{i,2} + \dot{V}_{i,3}\right]. \end{aligned} \tag{32}$$

From Assumption 2, we have $0 < f_i^L \le f_{u_i}$ which, in turn, yields $(1/f_{u_i}) \le (1/f_i^L)$ then, we can obtain the following upper bound for the time derivative of *V:*

$$\dot{V} \le \sum_{i=1}^{N}[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + \mathbf{e}_i^T P_i \mathbf{b}_i (u_i - u_i^*)$$

[DOR: 20.1001.1.26765810.1396.14.4.7.4]

$$+\frac{|\mathbf{e}_i^T P_i \mathbf{b}_i \| \sum_{j=1}^{N} h_{i,j} |}{f_i^L} + \dot{V}_{i,2} + \dot{V}_{i,3}] \quad (33)$$

From Assumption 3, (33) can be rewritten as

$$\dot{V} \le \sum_{i=1}^{N}\Bigg[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + \mathbf{e}_i^T P_i \mathbf{b}_i (u_i - u_i^*) + \frac{\left|\mathbf{e}_i^T P_i \mathbf{b}_i\right|}{f_i^L} \sum_{j=1}^{N} \eta_{i,j} (\mathbf{e}_j^T (t-\tau_{i,j}(t)) P_j \mathbf{b}_j) + \dot{V}_{i,2} + \dot{V}_{i,3} \Bigg] \quad (34)$$

By using inequality $xy \le (1/2)(x^2 + y^2)$, one obtains

$$\dot{V} \le \sum_{i=1}^{N}\Bigg[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + \mathbf{e}_i^T P_i \mathbf{b}_i (u_i - u_i^*) + \frac{1}{2}\sum_{j=1}^{N} \eta_{i,j}^2 (\mathbf{e}_j^T (t-\tau_{i,j}(t)) P_j \mathbf{b}_j) + \frac{1}{2}\sum_{j=1}^{N} \frac{\left(\mathbf{e}_i^T P_i \mathbf{b}_i\right)^2}{\left(f_i^L\right)^2} + \dot{V}_{i,2} + \dot{V}_{i,3} \Bigg] \quad (35)$$

$$= \sum_{i=1}^{N}\Bigg[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + \mathbf{e}_i^T P_i \mathbf{b}_i (u_i - u_i^*) + \frac{1}{2}\sum_{j=1}^{N} \eta_{i,j}^2 (\mathbf{e}_j^T (t-\tau_{i,j}(t)) P_j \mathbf{b}_j) + \frac{N\left(\mathbf{e}_i^T P_i \mathbf{b}_i\right)^2}{2\left(f_i^L\right)^2} + \dot{V}_{i,2} + \dot{V}_{i,3} \Bigg]$$

Substituting (20) and (21) into (35) yields

$$\dot{V} \le \sum_{i=1}^{N}\Bigg[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + (\mathbf{e}_i^T P_i \mathbf{b}_i)\tilde{\mathbf{B}}_i^T \Psi_{b_i}(\mathbf{z}_i) - (\mathbf{e}_i^T P_i \mathbf{b}_i)\varepsilon_i - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \hat{\mathbf{C}}_i^T \Psi_{c_i}(\mathbf{e}_i^T P_i \mathbf{b}_i) + (\mathbf{e}_i^T P_i \mathbf{b}_i) u_{i,R} + \frac{1}{2}\sum_{j=1}^{N} \eta_{i,j}^2 (\mathbf{e}_j^T (t-\tau_{i,j}(t)) P_j \mathbf{b}_j) + \dot{V}_{i,2} + \dot{V}_{i,3} \Bigg] \quad (36)$$

Using (30) to re-write (36) as

$$\dot{V} \le \sum_{i=1}^{N}\Bigg[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + (\mathbf{e}_i^T P_i \mathbf{b}_i)\tilde{\mathbf{B}}_i^T \Psi_{b_i}(\mathbf{z}_i) - (\mathbf{e}_i^T P_i \mathbf{b}_i)\varepsilon_i - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \hat{\mathbf{C}}_i^T \Psi_{c_i}(\mathbf{e}_i^T P_i \mathbf{b}_i) + (\mathbf{e}_i^T P_i \mathbf{b}_i) u_{i,R} + \frac{1}{2}\sum_{j=1}^{N} \eta_{i,j}^2 (\mathbf{e}_j^T (t-\tau_{i,j}(t)) P_j \mathbf{b}_j) + \frac{\sum_{j=1}^{N} \eta_{i,j}^2 (\mathbf{e}_j^T (t) P_j \mathbf{b}_j)}{2(1-\tau_{\max})} - \frac{1}{2(1-\tau_{\max})}\sum_{j=1}^{N} (1-\dot{\tau}_{i,j}(t)) \eta_{i,j}^2 (\mathbf{e}_j^T (t-\tau_{i,j}(t)) P_j \mathbf{b}_j) + \dot{V}_{i,3} \Bigg]$$

$$= \sum_{i=1}^{N}\Bigg[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + (\mathbf{e}_i^T P_i \mathbf{b}_i)\tilde{\mathbf{B}}_i^T \Psi_{b_i}(\mathbf{z}_i) - (\mathbf{e}_i^T P_i \mathbf{b}_i)\varepsilon_i - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \hat{\mathbf{C}}_i^T \Psi_{c_i}(\mathbf{e}_i^T P_i \mathbf{b}_i) + (\mathbf{e}_i^T P_i \mathbf{b}_i) u_{i,R} + \frac{\sum_{j=1}^{N} \eta_{i,j}^2 (\mathbf{e}_j^T (t) P_j \mathbf{b}_j)}{2(1-\tau_{\max})} + \dot{V}_{i,3} \Bigg] \quad (37)$$

Define $\left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \zeta_i(\mathbf{e}_i^T P_i \mathbf{b}_i) = \frac{\sum_{j=1}^{N} \eta_{i,j}^2 (\mathbf{e}_j^T (t) P_j \mathbf{b}_j)}{2(1-\tau_{\max})}$, and (37) becomes

$$\dot{V} \le \sum_{i=1}^{N}\Bigg[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + (\mathbf{e}_i^T P_i \mathbf{b}_i)\tilde{\mathbf{B}}_i^T \Psi_{b_i}(\mathbf{z}_i) - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \hat{\mathbf{C}}_i^T \Psi_{c_i}(\mathbf{e}_i^T P_i \mathbf{b}_i) + (\mathbf{e}_i^T P_i \mathbf{b}_i) u_{i,R} + \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \varepsilon_{M_i} + \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \zeta_i(\mathbf{e}_i^T P_i \mathbf{b}_i) + \dot{V}_{i,3} \Bigg] \quad (38)$$

Knowing that the function $\zeta_i(\mathbf{e}_i^T P_i \mathbf{b}_i)$ is smooth, a radial basis Neural Network can be utilized to approximate the following function

$$\zeta_i(\mathbf{e}_i^T P_i \mathbf{b}_i) = \mathbf{C}_i^{*T} \Psi_i(\mathbf{e}_i^T P_i \mathbf{b}_i) + \delta_i \quad (39)$$

where $\Psi_i(\mathbf{e}_i^T P_i \mathbf{b}_i) = \left[\psi_{i,1}(\mathbf{e}_i^T P_i \mathbf{b}_i), \psi_{i,2}(\mathbf{e}_i^T P_i \mathbf{b}_i), ..., \psi_{i,n_i}(\mathbf{e}_i^T P_i \mathbf{b}_i)\right]^T$ is the RBNN basis vector, and $\mathbf{C}_i^*$ is unknown vector of network weights and we estimate it with $\hat{\mathbf{C}}_i$, the term δ_i in (39) is called the NN approximation error satisfying $|\delta_i| \le \delta_{M_i}$, $\delta_{M_i} > 0$. Substituting (22) and (39) into (38) produces:

$$\dot{V} \le \sum_{i=1}^{N}\Bigg[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^2} + (\mathbf{e}_i^T P_i \mathbf{b}_i)\tilde{\mathbf{B}}_i^T \Psi_{b_i}(\mathbf{z}_i) - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \tilde{\mathbf{C}}_i^T \Psi_{c_i}(\mathbf{e}_i^T P_i \mathbf{b}_i) - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \tilde{\zeta}_i - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \tilde{\theta}_i + \dot{V}_{i,3} \Bigg] \quad (40)$$

[DOR: 20.1001.1.26765810.1396.14.4.7.4]

with employing time derivative $\dot{V}_{i,3}$, we have

$$\dot{V} \le \sum_{i=1}^{N}\Bigg[-\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} - \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^{\,2}} + \tilde{\mathbf{B}}_i^T\left(\Gamma_{b_i}^{-1}\dot{\hat{\mathbf{B}}}_i + \mathbf{e}_i^T P_i \mathbf{b}_i \Psi_{b_i}\right) + \tilde{\mathbf{C}}_i^T\left(\Gamma_{c_i}^{-1}\dot{\hat{\mathbf{C}}}_i - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right| \Psi_{c_i}\right) + \tilde{\zeta}_i\left(\frac{\dot{\hat{\zeta}}_i}{\gamma_{\zeta_i}} - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right|\right) + \tilde{\theta}_i\left(\frac{\dot{\hat{\theta}}_i}{\gamma_{\theta_i}} - \left|\mathbf{e}_i^T P_i \mathbf{b}_i\right|\right)\Bigg], \qquad (41)$$

By applying the adaptive rules (23)-(26), $\dot{V}$ can be rewritten:

$$\dot{V} \le \sum_{i=1}^{N} -\left[\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} + \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^{\,2}}\right]. \qquad (42)$$

To complete the proof, one needs to show that $\mathbf{e}_i^T Q_i \mathbf{e}_i / 2f_{u_i} + \mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i} / 2f_{u_i}^{\,2} > 0$ is positive, which is a direct result of Lemma 1. , Thus, $\mathbf{e}_i$, $\hat{\mathbf{B}}_i^T$, $\hat{\mathbf{C}}_i^T$, $\hat{\zeta}_i$ and $\hat{\theta}_i$ become bounded. Since (4) is ensured to be bounded, the functions $\eta_{i,j}(\mathbf{e}_j^T P_j \mathbf{b}_j)$ will also be bounded. Given (20)-(22), it is concluded that the entire variable on the right-hand side of (17) and $\dot{\mathbf{e}}_i$ are bounded. Moreover, since V is positive definite, we can conclude that:

$$\int_0^{\infty}\sum_{i=1}^{N}\left[\frac{\mathbf{e}_i^T Q_i \mathbf{e}_i}{2f_{u_i}} + \frac{\mathbf{e}_i^T P_i \mathbf{e}_i \dot{f}_{u_i}}{2f_{u_i}^{\,2}}\right] dt \le V(0) - V(\infty) < \infty \qquad (43)$$

Since the right side of (43) is bounded, $\mathbf{e}_i \in L_2$, by using Barbalat's lemma, we get to the conclusion that $\lim_{t\to\infty} \mathbf{e}_i(t) = 0$. This completes the proof.

4. Simulation

In this section, the proposed decentralized adaptive controller is applied to control two inverted pendulums connected by a spring [14]. The nonlinear equations which describe the motion of the pendulums are defined by

$$\begin{cases} \dot{x}_{1,1} = x_{1,2} \\ \dot{x}_{1,2} = \left(\frac{m_1 gr}{j_1} - \frac{kr^2}{4j_1}\right)\sin(x_{1,1}) + \frac{kr}{2j_1}(l-b) + \alpha_1 \frac{sat(u_1)}{j_1} + \frac{kr^2}{4j_1}\sin(x_{2,1}(t-\tau_{1,2}(t))) \\ y_1 = x_{1,1} \end{cases} \qquad (44)$$

$$\begin{cases} \dot{x}_{2,1} = x_{2,2} \\ \dot{x}_{2,2} = \left(\frac{m_2 gr}{j_2} - \frac{kr^2}{4j_2}\right)\sin(x_{2,1}) + \frac{kr}{2j_2}(l-b) + \alpha_2 \frac{sat(u_2)}{j_2} + \frac{kr^2}{4j_2}\sin(x_{1,1}(t-\tau_{2,2}(t))) \\ y_2 = x_{2,1}, \end{cases} \qquad (44)$$

where $\theta_1 = x_{1,1}$ and $\theta_2 = x_{2,1}$ are the angular displacements of the pendulums from vertical. The parameters $m_1 = 2kg$ and $m_2 = 2.5kg$ are the pendulum end masses, $j_1 = 0.5kg$ and $j_2 = 0.625kg$ are the moments of inertia, $k = 100N/m$ is the spring constant of the connecting spring, $r = 0.5m$ is the pendulum height, $l = 0.5m$ is the natural length of the spring, $\alpha_1 = \alpha_2 = 25$ are the control input gains and $g = 9.81m/s^2$ is the gravitational acceleration. The function $sat(\cdot)$ represents the actuators nonlinearity, which, in this simulation, is implemented by $\tanh(\cdot)$. The distance between the pendulum hinges is $b = 0.4m$. $b < 1$ indicate that the pendulum repel one another when both are in the upright position [28]. $\tau_{1,2} = \tau_{2,2} = 0.5(1+\sin(t))$, $\tau_{1,2}(t)$ and $\tau_{2,2}(t)$ satisfy $\dot{\tau}_{i,2}(t) \le \tau_{\max} < 1, \quad (i = 1,2)$. Here we will attempt to regulated the angular positions to zero, so that $e_i = -\theta_i$ [i.e., $x_i^d = 0, i = 1,2$].

To show the effectiveness of the proposed method, two controllers are studied for the purpose of comparison. We will first demonstrate how a simple decentralized PI controller

$$u_i = 20\left(e_i + \frac{1}{20}\int_0^t e_i \, d\tau\right), \qquad i = 1,2 \qquad (45)$$

would control the system. While the pendulums exhibit an undesirable response with relatively large oscillatory behavior due to the lack of damping, as shown in Figs. 1 and 2.

The decentralized adaptive controller based on the RBNN proposed in Sect. 3 is then applied to this system. The controller is taken as (21), where $\hat{\mathbf{B}}_i$, $\hat{\mathbf{C}}_i$, $\hat{\theta}_i$ and $\hat{\zeta}_i$ are updated by adaptive rules (23-26), and $u_{i,k} = 20(e_i + 1/20\int_0^t e_i dt)$. The RBNN structure is used as given in (22). The radial basis functions $\psi_{b_i}(\mathbf{z}_i)$ are chosen as $\psi_{b_{i,q}} = \exp(-\|z_i - \zeta_{b_{i,q}}\|^2 / \sigma_{b_{i,q}}^2)$, where $\zeta_{b_{i,q}}$ and $\sigma_{b_{i,q}}$ are the centers and size of influences of the basis function, respectively $(i = 1,2, q = 1,2,...,K_i)$. The input vector for the RBNN basis ψ_{b_i} is

[DOR: 20.1001.1.26765810.1396.14.4.7.4]

$\mathbf{z}_i = [x_{i,1}, x_{i,2}, v_i]^T$. The RBNN nodes are chosen as $K_i = 100 (i = 1,2)$, with the centers $\zeta_{b_{i,q}} = [\zeta_{b_{i,q1}}, \zeta_{b_{i,q2}}, \zeta_{b_{i,q3}}]^T$ evenly spaced between [-1,1], [-5,5], and [-35,35], respectively, and the size of influences $\sigma_{b_{i,q}} = 0.5, (i = 1,2, q = 1,2,...,100)$. The basis function $\psi_{c_i} (\mathbf{e}_i^T P_i \mathbf{b}_i)$ are chosen as $\psi_{c_{i,q}} = \exp(-|\mathbf{e}_i^T P_i \mathbf{b}_i - \zeta_{c_{i,q}}|^2 / \sigma_{c_{i,q}}^2)$, where $\zeta_{c_{i,q}}$ and $\sigma_{c_{i,q}}$ are the centers and size of influences of the basis function, respectively $(i = 1,2, q = 1,2,..., D_i)$. The input vector for the RBNN basis ψ_{c_i} is $\mathbf{e}_i^T P_i \mathbf{b}_i$. The RBNN nodes are chosen as $D_i = 5 (i = 1,2)$ nodes, with centers evenly space between [-1,1] and the size of influences $\sigma_{c_{i,q}} = 0.5$ $(i = 1,2, q = 1,2,...,25)$. The initial RBNN weights $\hat{\mathbf{B}}_i(0), \hat{\mathbf{C}}_i(0), \hat{\theta}_i(0)$ and $\hat{\theta}_i(0)$ are simply set to zeros. The controller parameters are taken as $f_i^L = 1, \Gamma_{b_i} = \Gamma_{c_i} = 100,$ and $\gamma_{\theta_i} = \gamma_{\zeta_i} = 5.$ Figs. 3 and 4 show the simulation results for the designed controller and illustrate that, after a short transient period, the states very closely track the given trajectories. Comparing the results in Figs. 1-4, it can be concluded that with the proposed method, the weights of neural networks have fast convergence and the performance is more satisfactory compared to PI counterpart. Figs. 5 and 6 also show the history of the control input u_i, $i = 1,2$ under proposed controller.

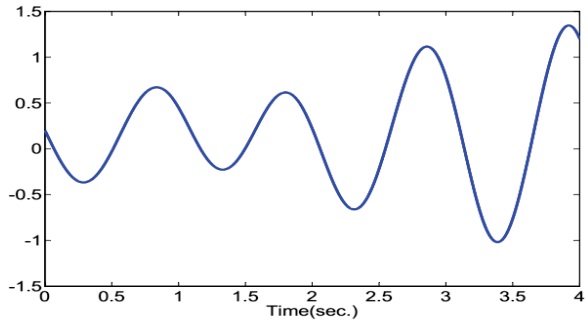

Fig. 1. PI controller for the first subsystem.

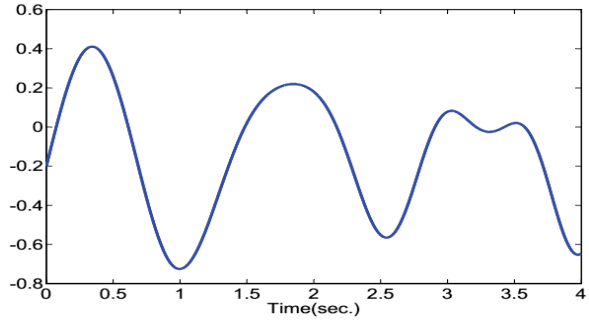

Fig. 2. PI controller for the second subsystem.

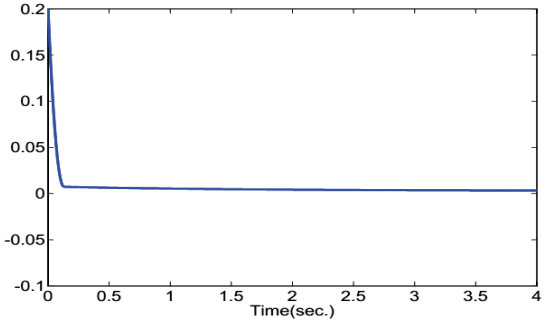

Fig. 3. Performance of the proposed controller for the first subsystem.

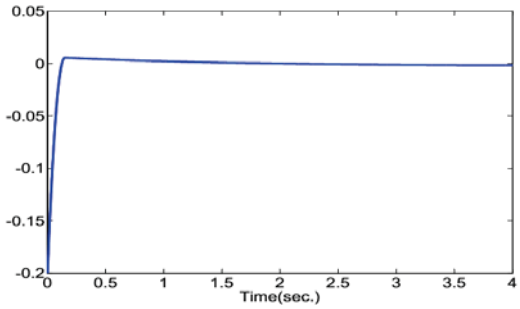

Fig. 4. Performance of the proposed controller for the second subsystem.

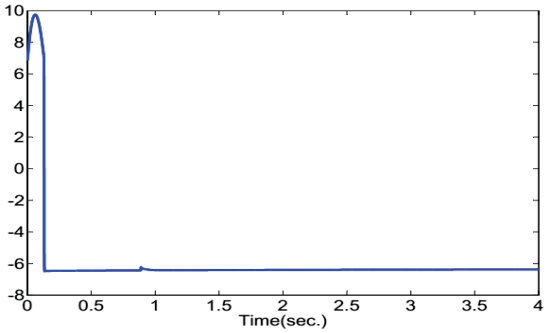

Fig. 5. The control input $\mathbf{u}_1$ for the first subsystem.

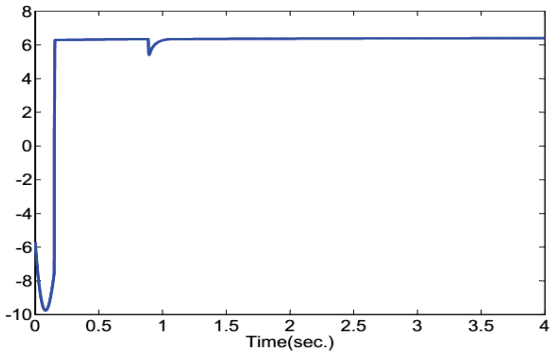

Fig. 6. The control input $\mathbf{u}_2$ for the second subsystem.

5. Conclusion

In this paper, a new decentralized adaptive RNBB control was developed for a class of large scale nonlinear non-affine systems with unknown nonlinear time-varying

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delay interconnections. Using the stability analysis of Lyapunov-Krasovskii functional method, the asymptotically stability of the closed-loop system was proved. Finally, simulation results confirmed the good performance of the proposed controller compared to traditional PI controller.

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