Comprehensive Electromechanical Analysis of MEMS Variable Gap Capacitors

Hooman Nabovati[1], Khalil Mafinezhad[1,2], Aydin Nabovati[3], Hosseyn Keshmiri[2]

1- Department of Electrical Engineering, Sadjad University, Mashhad, Iran
2- Department of Electrical Engineering, Ferdowsi University, Mashhad, Iran
3- Department of Mechanical Engineering, University of New Brunswick, Fredericton, Canada
nabovati@sadjad.ac.ir

Abstract:

This paper presents a comprehensive case study on electro-mechanical analysis of MEMS[1] variable capacitors. Using the fundamental mechanical and electrical equations, static and dynamic behaviors of the device are studied. The analysis is done for three different modes, namely: dc (static mode), small signal ac and large signal regime. A complete set of equations defining dynamic behavior of the MEMS, and an ac small signal equivalent circuit are presented. The mathematical models are defined and examined by the Matlab Simulink and a complete set of simulation results is reported for various cases separately. The results of this study would be useful in design and analysis of the MEMS based circuits which have some kind of mechanical dynamic action. Some examples of such devices may include VCO[2]s, frequency modulators, tunable filters and parametric effect circuits.

Keywords: MEMS, Variable capacitors, Electro-mechanical modeling

1. Introduction

The increasing demand for light weight and miniaturized cell phones, laptops, global positioning system receivers and remote sensors, has spawned an explosive growth in the wireless technology in the recent decade. As the demand for smaller devices and more efficient use of allocated spectral frequency range increases, much more capable implantation technologies are required. In recent years, MEMS technology has begun to be used in wireless communication systems to improve performance of the existing devices based on the structural and operational principles.

At present the ultimate miniaturization of super heterodyne transceivers is mainly restricted by the need for numerous off-chip frequency selective passive components such as variable capacitors and inductors. Variable capacitor is a basic component of a voltage controlled oscillator (VCO) used in frequency synthesizer, which generates the local oscillator signals. They can be also used in tunable filters, frequency modulators and parametric effect circuits as well.

Recent demonstration of the voltage tunable capacitors comprised of micro-machined, movable metal plates offer substantial improvements over varactor diodes. Compared with the solid state varactors, micro-machined variable capacitors have the advantage of lower loss, lower noise, higher quality factor and potentially greater tuning range [1-4].

The recent applications of the MEMS technologies in the voltage tunable capacitors are using two kinds of methods, namely: the electro-thermal method and the electrostatic method. In electrostatic method, capacitance is tuned by varying the distance between two parallel flat plates using an electrostatic force caused by a bias voltage. The desired capacitance accomplished by the fast tuning and small space, but the theoretical tuning range is limited to only 150% of the reference value [5]. Among all the MEMS tunable capacitors developed so far, the parallel plate configuration with electrostatic actuation is the most commonly used [4, 5].

In the present work, Electro-mechanical behavior of a MIM[3] MEMS variable capacitor has been analytically studied. Dynamic analysis of the structure is useful for studying the transient response of the MEMS. Moreover some applications of the MEMS varactor devices, such as modulators, frequency multipliers and parametric-effect amplifiers are based on the dynamic behavior of the MEMS device. The analysis has been done using the classical electro-mechanical equations and verified and examined by the Matlab modeling capabilities. Beside, a general dynamic model for the MEMS variable capacitors has been presented in dc, small signal and large signal regime separately. All the models have been examined by the Matlab Simulink, and the simulation results are in very good agreement with the analytical calculations.

[1] - Micro Electro Mechanical System
[2] - Voltage Controlled Oscillator
[3] - Metal Insulator Metal

[Downloaded from jiaeee.com on 2026-11-20]

PDF created with pdfFactory Pro trial version www.pdffactory.com

2. MEMS Varactor Specification

Figure (1) shows the studied structure schematically, [6]. The top plate is moving and suspended by four oblique cantilever beams, bottom plate is fixed. Oblique beams act as a spring with higher elastic constant in compare with the normal arms which results in a higher resonance frequency and a broader tenability range.

Figure (2) shows the six layers of the structures which are used in the MUMPS[4] technology.

The capacitor characterization was captured using the electromagnetic simulation of the structure. The full wave electromagnetic simulation of the capacitor was done using the MEM Research EM3DS 6.1 software [7, 8]. After a full wave analysis, the y parameters of the structure were extracted in a wide frequency range for different distances between the capacitor plates.

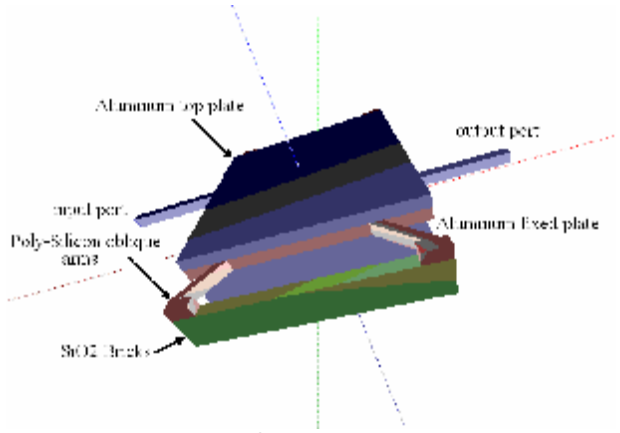


Fig. 1: Schematics of the MEMS variable capacitor

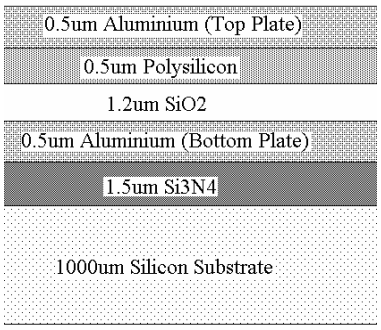


Fig. 2: Profile of The MEMS Layers

A simple equivalent circuit for the structure consists of a MEMS intrinsic capacitor in series with a resistance, which models the structure dissipation. Using the results of full wave electromagnetic analysis, the equivalent circuit parameters were extracted and scaled according to the gap distance; the calculated y parameters in the last step were used for this purpose. The procedure is straightforward and is presented in equations (1) and (2).

$$C = \frac{\left|\mathrm{Im}(y_{12})\right|}{2p\,f} \quad (1)$$

$$R_S = \frac{R_p}{1+Q_c^2} \quad (2)$$

where, $Q_c = R_p C w$ is the capacitor quality factor, and R_p indicates the MEMS equivalent parallel Resistor, which was calculated using the following equation;

$$R_p = \frac{1}{\left|\mathrm{Re}(y_{12})\right|} \quad (3)$$

The values of the equivalent circuit elements were calculated for various device dimensions. According to these calculations, a behavioral intrinsic model was extracted which defined the C and R_S as a function of the gap distance. At this stage, it was assumed that like a classical parallel plate capacitor, the capacitance is a linear function of the inverse of the plate distance (1/d). Because of parasitic effects, e.g. fringing, the intrinsic capacitor has a 29.5fF offset when 1/d tends to zero. Equation (4) presents the capacitance variation as a function of the gap distance for the structure under study. The parameter extraction algorithm is realized by the MATLAB 7.0 software from the Mathworks. The variation of R_S with the gap distance is almost negligible.

.

$$C(fF) = 29.5 + \frac{462.8}{d(mm)} \quad (4)$$

The results verified the structure capacitance can be calculated as a classic parallel plate capacitor.

3. Principles of the Operation

As it is shown in figure (3), dynamic analysis of the MEMS variable capacitor is based on the suspended resonator model. The system is described by the following differential equation [9-12],

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = f_e(x,t) \quad (5)$$

where m, b, k, and f_e are mass of the movable top plate, damping coefficient, spring constant and electrical force, respectively. These parameters are related to the physical specifications of the structure and for a rectangular plate with 4 oblique cantilever arms, we have,

$$m = \rho abt \quad (6)$$

[4] - Multi User MEMS Process

Journal of Iranian Association of Electrical and Electronics Engineers – Vo1.4- No.2- Fall and Winter 2007
[Downloaded from jiaeee.com on 2026-11-20]
PDF created with pdfFactory Pro trial version www.pdffactory.com

and,

$$k = \frac{12EI}{l^3} \qquad (7)$$

where ρ, a, b, t, E, l and I are density of the plate material, length, width and thickness of capacitor plate, Young's module of the cantilever beam's material, the cantilever beam length and moment of inertia of the cantilever beam. The values of these parameters are summarized in table (1) for the proposed structure.

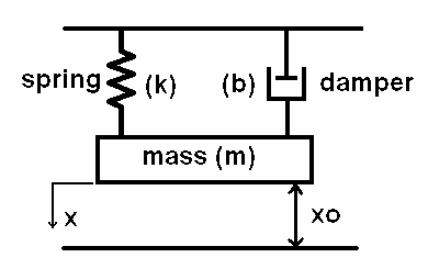


Fig. 3: dynamic mechanical model for MEMS variable capacitor

The damping coefficient, b, expresses the energy dissipation in the system by airflow force, squeeze force and internal friction. It has been related to the mechanical quality factor, Q, and will be discussed in next chapters.

Table 1: Physical parameters for the capacitor

Physical Specification						
ρ (g/cm^3)	a (μm)	b (μm)	t (μm)	l (μm)	E (GPa)	I (μm^3)
2.32	200	200	0.5	42	170	2.9×10^{-7}

4. DC Analysis

In static mode, when a DC bias voltage is applied to the plates, an electrostatic force will be generated between the two plates which forces the top plate to move toward the fixed one, until equilibrium between the electrostatic and mechanical forces -exerted by the oblique arms which act as four springs- is achieved. According to the previous chapter, neglecting the fringe effect, the capacitance of the structure, which is formed between two plates, can be written as:

$$C = \varepsilon \frac{A}{x_0 - x} \qquad (8)$$

where x, x_0, A, ε are the top plate displacement, initial gap size, plate surface area and permittivity of air gap. x_0-x is the minimum gap size between two plates.
When a DC bias voltage, V, is applied, the stored electrical energy in the capacitor is calculated as follows:

$$E = \frac{1}{2}CV^2 = \frac{\varepsilon A V^2}{2(x_0 - x)} \qquad (9)$$

Electrostatic force between two plates is then calculated as follows:

$$F_e = -\frac{\partial E}{\partial (x_0 - x)} = \frac{1}{2}\frac{\varepsilon A V^2}{(x_0 - x)^2} \qquad (10)$$

The static mechanical force exerted to the top plate due to the deflection of the four oblique beams, can be calculated using the equation (1); in static analysis it is simplified as:

$$F_m = kx = \frac{12EI}{l^3}x \qquad (11)$$

Regarding static equilibrium between the electrostatic and mechanical forces, one can write:

$$\frac{1}{2}\frac{eAV^2}{(x_0 - x)^2} = \frac{12EI}{l^3}x \qquad (12)$$

Using equation (12), the plate displacment can be calculated as a function of the bias voltage and the structure physical parameters.
It is important to note that the corresponding value of V at x = x_0/3 is the critical point and is called pull-in voltage. If V is increased beyond this limit, no equilibrium can be achieved and the top plate will move toward the bottom one until they snap into contact; this phenomenon is called the pull-in effect. Therefore according to equation (8), theoretically the maximum capacitance of the variable capacitor is 150% of its initial value at V=0.
Figure (4) shows the distance of two plates versus the applied bias voltage. The initial distance was assumed to be equal to 1.2μm and the pull-in effect occurred in 2.88V; at this point the distance between two plates reached the critical value of 0.8μm.

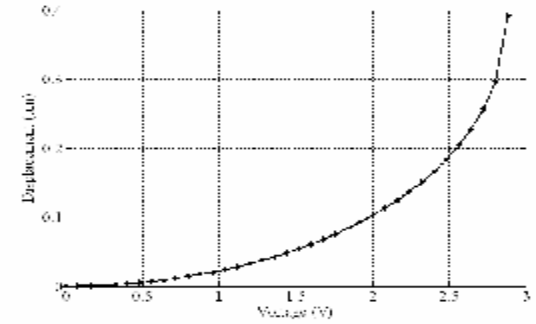


Fig. 4: displacement of the plates verses bias voltage

[Downloaded from jiaeee.com on 2026-11-20]
PDF created with pdfFactory Pro trial version www.pdffactory.com

5. Small Signal Analysis

In the linear analysis or small signal regime, it was assume the displacement, x, was small compared to the initial gap distance. The structure was driven by a small ac voltage v_{ac}, superimposed on the dc bias V, which induced the small displacement variations. The dynamics of the resonator was approximately determined by the equation (5), where f_e, the small signal electrostatic force expressed by the following equation [13-16],

$$f_e = \left(\frac{\partial f_e}{\partial V}\right) v_{ac} = \frac{\varepsilon A V v_{ac}}{x_0^2} = 2F_e\left(\frac{v_{ac}}{V}\right) \qquad (13)$$

In this equation, F_e is the static electric force and v_{ac} was assumed to be a small signal sine voltage so the equation (5) can be solved to find the displacement in the phasor form, as follows:

$$\hat{x} = \frac{2F_e}{V}\frac{\hat{v}_{ac}}{k + j\omega b - m\omega^2} \qquad (14)$$

The structure current could be considered as a nonlinear capacitor, in the form of:

$$i = \frac{d}{dt}(cv) = \frac{dc}{dt}\cdot v + c\cdot\frac{dv}{dt} \qquad (15)$$

where c indicates the time dependant nonlinear capacitor. Equation (8) can be rewritten in differential form as;

$$\frac{dc}{dt} = \frac{dc}{dx}\times\frac{dx}{dt} \approx j\omega\frac{\varepsilon A}{x_0^2}\hat{x} \qquad (16)$$

where $\hat{x}$ is the phasor of the displacement. Considering small variation in v and c, we can replace c and v by the following equations,

$$v = V + v_{ac} \approx V \qquad (17)$$

and;

$$c = C + c(t) \approx C \qquad (18)$$

where c(t) indicates the variations in structure capacitance and C is the static MEMS capacitance as,

$$C = \frac{\varepsilon A}{x_0} \qquad (19)$$

Therefore the MEMS current can be calculated as follows:

$$i \approx CV\hat{v}_{ac} + V\hat{c} = jCw\hat{v}_{ac} + \frac{jw\hat{v}_{ac}}{kx_0^2/C^2V^2 + jwbx_0^2/C^2V^2 - w^2mx_0^2/C^2V^2} \qquad (20)$$

An equivalent circuit model can be defined for the structure [12]. The equivalent circuit model is presented in figure (5), and its admittance can be determined as follows:

$$Y = jwC_0 + \frac{1}{jwL_1 + R_1 + 1/jwC_1} \qquad (21)$$

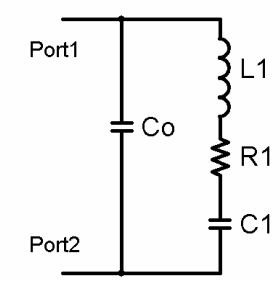


Fig. 5: Equivalent circuit model for small signal analysis

Circuit parameters can then be calculated by the analogy to the equivalent circuit equations. Effect of the mechanical properties of the structure is shown in the following equations.

$$C_1 = \frac{C^2V^2}{kx_0^2} \qquad (22)$$

$$R_1 = \frac{bx_0^2}{C^2V^2} \qquad (23)$$

$$L_1 = \frac{mx_0^2}{C^2V^2} \qquad (24)$$

$$C_0 = C \qquad (25)$$

In the present study the mechanical quality factor and the dc bias voltage were assumed to be 20 and 1V. Using these values, the equivalent circuit parameters were calculated as 10.5fF, 3.09GΩ, 767H and 295fF for C_1, R_1, L_1 and C_0, respectively. The resonance frequency and the electro-mechanical quality factor of the MEMS are defined as,

$$w_0 = \frac{1}{\sqrt{L_1C_1}} = \sqrt{\frac{k}{m}} \qquad (26)$$

$$Q = \frac{1}{w_0R_1C_1} = \frac{w_0L_1}{R_1} = \frac{w_0m}{b} \qquad (27)$$

[Downloaded from jiaeee.com on 2026-11-20]
PDF created with pdfFactory Pro trial version www.pdffactory.com

When the mechanical parameters of the structure are fully determined, there is a straightforward procedure to find out the equivalent circuit model.

An alternative routine method is to use the electromagnetic simulation. In this method the structure geometry and its electromagnetic factors are defined in a full wave electromagnetic simulator, e.g. the Ansoft HFSS or the MEM Research EM3DS. The simulation results would be small signal parameters of the network, such as S or Y parameters. These parameters can be examined through a wide frequency range and their resonance frequency and quality factor could be compared with the electro-mechanical analysis.

The equivalent circuit parameters can then be determined directly as summarized in following equations,

$$R_1 \approx \left.\frac{1}{\mathrm{Re}(y_{12})}\right|_{w=w_o} \quad (28)$$

$$C_1 \approx \frac{1}{w_o Q R_1} \quad (29)$$

$$L_1 \approx \frac{1}{C_1 w_o^2} \quad (30)$$

$$C_o \approx \left.\frac{y_{12}}{w}\right|_{w>>w_o} \quad (31)$$

The small signal electro-mechanical model of the structure was simulated in the Matlab Simulink environment. In this simulation, sinusoidal signals of the amplitude of 100mV and 200 mV were applied to the MEMS structure. It could be seen that if the exerted signal amplitude was less than 140mV, it was restricting the displacement of the plates to less than 10% of x_o. Figure (6) represents the MEMS capacitance. It can be seen that in small signal analysis, capacitance variation is almost a sinusoidal function. Using the fast Fourier transform to determine the capacitor variation spectrum resulted that the capacitance distortion was almost insignificant. The frequency of input signal was selected as 59 KHz which was equal to the resonance frequency of the structure.

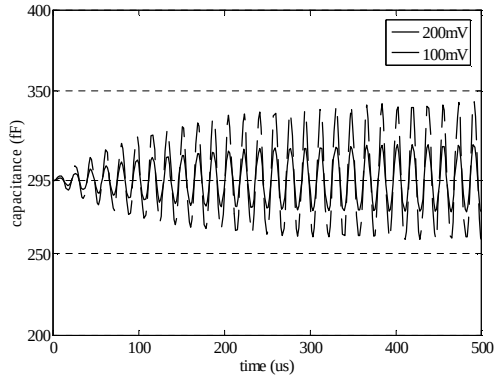


Fig. 6: Capacitance variation versus time with 100-200mV small signal excitation

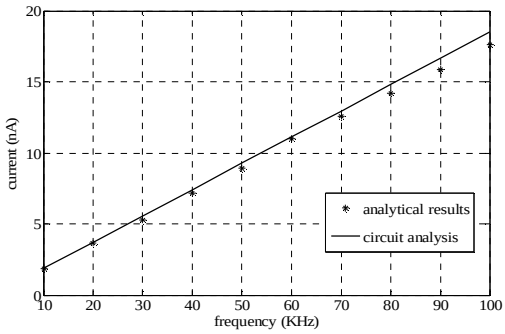


Fig. 7: Capacitor current with 100mV sinusoidal excitation versus frequency (results of analytical calculations and circuit analysis)

Figure (7) depicts a comparison between the current calculated from the electrical equivalent circuit and the current obtained from the analytical solution using equation (15) over a wide frequency range of 10-100 kHz. The Electrical simulation results are in very good agreement with the analytical results.

Since the structure current is small, the effect of the MEMS series resistance was neglected.

6. Large Signal Analysis

When a large signal is applied to the MEMS, the structure acts completely as a nonlinear time variant capacitor. It is useful to investigate this behavior because of its applications in the parametric effect circuits, [17].

With increasing the applied voltage, the structure current can be high enough to make a considerable voltage drop on any serial resistor including the source impedance and the MEMS series resistor. If v_a indicates the applied voltage and R_s stands for the total serial resistance, by using the equation (15) voltage of the MEMS can be calculated by the following equation,

$$v = v_a - R_s \frac{d(cv)}{dt} \quad (32)$$

As it was mentioned before, in this regime the MEMS behaves like a nonlinear time variant capacitor; therefore the electrical stored energy is calculated as,

$$E(t) = \int vidt = \int v \frac{d(cv)}{dt} dt = \frac{3}{2} cv^2 \quad (33)$$

and the electrical force can be determined as,

$$f_e = \frac{dE(t)}{dx} \quad (34)$$

using equation (8), the electrical force is presented in the following form:

[Downloaded from jiaeee.com on 2026-11-20]
PDF created with pdfFactory Pro trial version www.pdffactory.com

$$f_e = \frac{3}{2}\frac{eA}{(x_0 - x)^2}v^2 \qquad (35)$$

The large signal model can be defined using equations (32), (35) and the MEMS dynamic mechanical equation, equation (5). The model has been implemented in the Matlab Simulink environment. For the time domain transient analysis, the input voltage was supposed to be a sinusoidal voltage of 1V amplitude and frequency of 59 kHz.

Figure (8) shows the structure capacitance as a function of time using the large signal analysis. It can be seen that the MEMS capacitance is a nonlinear function of the input signal and does not vary sinusoidal. To describe the nonlinear behavior of the device, the Fourier expansion in the form of equation (15) was used to describe the capacitance as follows:

$$c(t) = a_0 + \sum_n a_n \cos(nwt) + b_n \sin(nwt) = a_0 + \sum_n g_n \cos(wt + j_n) \qquad (36)$$

Determining c(t) from the electromechanical simulations, the coefficients a_n and b_n or γ_n and φ_n were evaluated by numerical methods. The capacitor variation spectrum is presented in figure (9). Using these calculations, γ_1 and γ_2 were evaluated to be equal to 33fF and 43fF for this level of excitation.

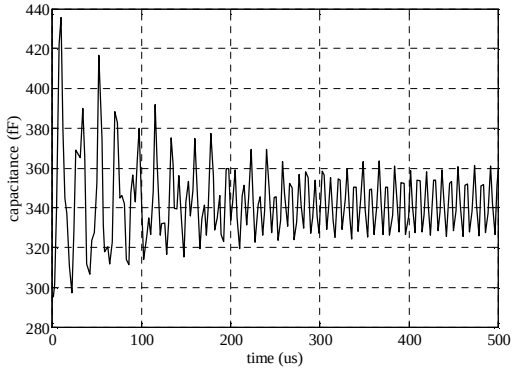


Fig. 8: capacitance variation when a 2V sinusoidal large excitation is applied

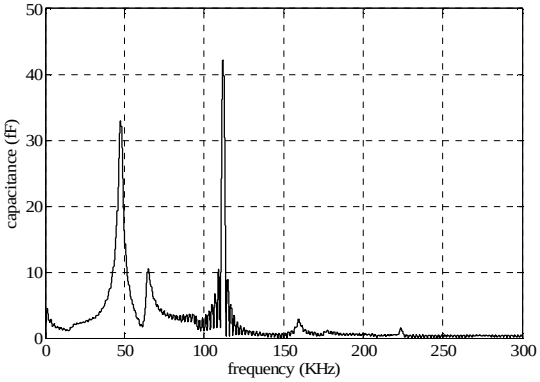


Fig. 9: capacitance variation spectrum

7. Conclusion

A complete case study on the electro-mechanical behavior of MEMS variable gap capacitors was presented. The device modeling was captured in three different modes; namely: dc, small signal ac and large signal stimulation. For each mode, a comprehensive set of electro-mechanical equations were presented which were implemented in the Matlab Simulink environment. An ac equivalent circuit model, was also elaborated which described the device electro-mechanical dynamic behavior according to its geometry and the physical characteristics of the proposed structure. The results of this study are useful in design and analysis of the MEMS based circuits which include mechanical dynamic behavior; e.g. VCOs, frequency modulators, tunable filters and parametric effect circuits

Acknowledgement

The authors would like to thank Ms. Gelareh-Veysi for her assistance and Professor Marco Farina from the MEM Research for his guidance and providing EM3DS tool to carry out this work. The supports of the Iran Telecommunication Research Center and the Sadjad Research Center are also acknowledged.

References

[1] C.T.Nguyen, "Communication Application of Miroelectromechanical Systems", Proceedings, 1998 Sensors Expo, San Jose, CA, pp. 447-455, May 1998.

[2] L.P.B. Katehi, J.F. Harvey, E. Brown, "MEMS and Si Micromachined Circuits for High-Frequency Applications", IEEE Transaction on Microwave Theory and Techniques, pp. 858-866, Vol. 50, No. 3, March 2002.

[3] H.J. De Los Santos, RF MEMS Circuit Design, Artech House, Boston, 2002.

[4] N. Bushyager, M.M. Tentzeris, M.M. Gatewood, L. DeNatale, "A novel adaptive approach to modeling MEMS tunable capacitors using MRTD and FDTD techniques", Microwave Symposium Digest, 2001 IEEE MTT-S International, pp. 2003 –2006, Vol. 3 , 20-25 May 2001.

[5] J. Zou, C. Liu, J. Schutt-Aine, J. Chen, S. Kang, "Development of a wide tuning range MEMS tunable capacitor for wireless communication systems", Electron

[Downloaded from jiaeee.com on 2026-11-20]

PDF created with pdfFactory Pro trial version www.pdffactory.com

Devices Meeting, IEDM Technical Digest International, pp. 403-406, Dec. 2000.
[6] H. Nabovati, K. Mafinezhad, H. Keshmiri, A. Nabovati, "Design and Simulation of an Oblique Suspender MEMS Variable Capacitor", Scientia Iranica Journal, Vol. 12, No 1, 2005.
[7] M. Farina, T. Rozzi, "A 3-D integral equation-based approach to the analysis of real-life MICs application to microelectro-mechanical systems", IEEE transaction on microwave theory and techniques, Vol. 49, No. 12, , pp. 2235-2240, December 2001.
[8] EM3DS user manual, Release 1.5, MEM Research, May 2003.
[9] Y. Kim, S.G. Lee, S. Park, "Design of the Two-Movable Plate Type MEMS Voltage Tunable Capacitor" Technical proceeding of 2002 of the 2002 international conference on modeling and simulation of Microsystems, Nanotech, 2002.
[10] F.P. Beer and E.R. Johnston, Mechanics of Materials, McGraw Hill, 1992.
[11] R.N. Simons, Coplanar waveguide circuits, components and systems, New York, NY, John Wiley & Sons, 2001.
[12] H. Eskelinen, P. Eskelinen, Microwave component mechanics, Norwood MA, Artech House, 2003.
[13] J.P. Raskin, A.R. Brown, T. Khuri-Yakub, G.M. Rebeiz, "A novel parametric effect MEMS amplifier", Journal of microelectromechanical systems, Vol. 9, No. 4, pp. 528-536, Dec 2000.
[14] E.M. Abdel-Rahman, A.H. Nayfeh, M.I. Younis, "Dynamics of an electrically actuated resonant microsensor", International Conference on MEMS, NANO and Smart Systems Proceedings, pp. 188-196, July 2003.
[15] C. Mandelbaum, S. Cases, D. Bensaude, L. Basteres, P. Nachtergaele, "Behavioral Modeling and Simulation of Micromechanical Resonator for Communications Applications", IEEE Conference on Design, Test, Integration and Packaging of MEMS and MOEM, pp. 21-26, May 2003.
[16] A. Cruaul, P. Nicole, G. Lissorgues, C. Tassed, "Influence of RF signal power on tunable MEMS capacitors", IEEE 33rd European Microwave Conference, pp. 663-666, 2003.
[17] H. Nabovati, K. Mafinezhad, H. Keshmiri, "Design and Simulation of Low Noise Upconverter Using MEMS Variable Capacitor", ICEE2007 Proceeding, 2007.

[Downloaded from jiaeee.com on 2026-11-20]
PDF created with pdfFactory Pro trial version www.pdffactory.com