A New Approach for Accurate Pricing of Reactive Power and Its Application to Cost Allocation in Deregulated Electricity Markets

S. Hasanpour[1] **R. Ghazi**[2] **M. H. Javidi**[3]

1- MSc, Department of Electrical Engineering, Sadjad Institute of Higher Education, Mashhad, Iran
s_hassanpour@sadjad.ac.ir

2- Professor, Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
rghazi@um.ac.ir

3- Professor, Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
h-javidi@ferdowsi.um.ac.ir

Abstract:

Reactive power management plays an essential role in the secure operation of the power system as an ancillary service. Although in electricity markets, the particular attention is paid to active power, the reactive power also plays an important on total generation costs of electricity. On the other hand, as it is mainly confined to local consumption, to avoid market power and maintain the secure operation of a power system, accurate reactive power pricing and cost allocation methods are essential. It has been a challenging problem during the past decade. However, most methods proposed so far for reactive power pricing, are essentially based on empirical approximations. In this paper a new method for reactive power cost allocation is proposed. The method is based on the calculation of accurate cost imposed on generators supporting reactive power. The proposed method is fair, accurate and realistic and it can be formulated very easily. Furthermore, a new approach based on tracing algorithm is proposed for pricing of reactive power which considers the cost of both active and reactive losses allocated to each generator and cost of capacitor banks. To validate the performance of the proposed method, it is applied to both 9-bus and 30-bus IEEE test systems.

Keywords: Reactive power pricing, Cost allocation, Tracing algorithm, Restructuring

Submission date: Jun. 6, 2008

Acceptance date: Nov. 22, 2010

Corresponding author : S, Hasanpour

Corresponding Author's Address: Department of Electrical Engineering, Sadjad Institute of Higher Education, Mashhad, Iran

1. Introduction

Correct reactive power management is needed to provide the secure and reliable operation of power systems. In vertically integrated power system its cost is usually recovered using approximate methods. In some systems the cost is included in the price of active power while in some other systems; the power factor is used as a penalty factor to compensate the cost.

In a restructured power system it is considered as an ancillary service and priced separately. An equitable pricing of such a service can lead to market liquidity which in turn results in approaching the optimal condition.

Many investigations have been carried out for appropriate pricing of reactive power [1-10]. Some of these methods utilize various search techniques such as genetic and ant colony algorithms for pricing [3]. Some other methods focus on formulating reactive power pricing [4, 5]. Muchayi et al. in [6] have presented a survey on some of the reactive pricing algorithms. Ketabi et al. in [7] have proposed a pricing technique based on minimization of the generator active and reactive power production and capacitor bank costs using the ant colony algorithm. Cost allocation of reactive power by modified Y-bus matrix method has been reported by Chu et al. in [8]. Rider et al. in [9] have proposed a nonlinear reactive power pricing method. They have presented the total cost of reactive power production as a nonlinear model which is solved by modified predictor-corrector interior-point method. Pricing of real and reactive power as bundled products in synchronous machine has been investigated in [10]. Xie et al. have proposed a method for active and reactive power pricing using interior point nonlinear optimization [11]. In their approach, spot prices are decomposed into different components reflecting various ancillary services. Seifossadat et al. have presented the sequential linear programming method to solve the reactive power pricing problem in [12].Chung et al. have proposed a method for cost-based reactive power pricing in which the cost of reactive power production by generators and capacitors are minimized [13]. Singh et al. in [14] have presented a method for active and reactive power allocation of thermal units in which the operation cost, impacts on environment and active power loss are minimized. A methodology for calculation of cost of reactive power by generators, synchronous condenser and static reactive power sources has been reported by Deksnys et al. [15]. Also a methodology for reactive power cost allocation based on modified power flow tracing method has been proposed by Tiawari et al. in [16]. Ro has presented the reactive charging scheme composed of recovering capital cost and operational cost [17].

The cost of generator reactive power consists of two components: fixed costs or investment costs and variable costs. Variable costs in turn consist of operating costs (including fuel and maintenance costs) and the opportunity cost. The latter cost results from reduction of its active power generation.

In this paper, a new method is proposed for reactive power pricing in a deregulated power market. This method utilizes the accurate relation between active and reactive power and then define a quadratic cost function. In our proposed approach, various components of reactive power cost including investment, operation and opportunity cost have been considered.

Some reactive power pricing methods only consider the cost of active losses which is attributed to one generator (slack bus), in the optimization problem [13]. In the presented method the cost of both active and reactive power losses are considered. In addition, the contribution of each generator in losses is determined using the tracing method. Then, the total costs i.e. the cost of active and reactive power production and the cost of active and reactive losses and cost of capacitor banks are minimized through an optimization process. To show the credibility of the proposed approach, it has been applied to IEEE 9 and 30 bus test systems.

This paper is organized in 5 sections. The procedure of cost allocation method is introduced in Section2. In Section3, the analysis of cost for reactive power support and reactive power pricing is discussed. The simulation results along with necessary comparison are appeared in Section 4. The conclusions that can be drawn from this paper are presented in Section 5.

2. Reactive Power Cost Allocation

Cost of reactive power is conventionally calculated by using the following empirical quadratic expression:

$$cost(Q) = .05 b_p Q^2 \qquad (1)$$

It should be noted that the active and reactive power of each generator are essentially bundled with each other. In equation 2 only the operational cost of reactive power is considered.

In [11], a second order polynomial is used for the cost of reactive power, in which the constant coefficients a, b and c are approximated to be one tenth of those for the cost of active power. In [4, 7], the triangular approach is proposed for cost evaluation of the reactive power based on the triangular relationship between the active and reactive power. In this triangular approach, the cost of reactive power is formulated by

$$cost(Q) = a''Q^2 + b''Q + c'' \qquad (2)$$

where, a'', b'', c'' are constants depending on the value of the power factor ($\cos\theta$) and given by

$$\begin{aligned} a'' &= a_p \sin^2\theta \\ b'' &= b_p \sin\theta \\ c'' &= c_p \end{aligned} \qquad (3)$$

This formulation is basically similar to the active power cost formulation, except that the active power is replaced by reactive power using the triangular relationship. As the investment for generators is based on the optimal solution for active production costs,

Journal of Iranian Association of Electrical and Electronics Engineers - Vol.7- No.2- Fall & Winter 2010

employing the same formula for reactive power cost will lead to wrong fixed costs for reactive power.
Therefore, these methods can be considered as approximation methods for reactive power pricing and may be valid only for a special range of reactive power production.
The present paper proposes a new method for formulation of reactive cost allocation. Attempt has been made to formulate the equation for cost of reactive power by a quadratic function as below. Meanwhile, it should be noted that as the capacity of each generator is limited by its armature current, field current and the under-excitation mode of operation, the production of reactive power may require a reduction in real power. So, depending on the operating point of the generator, the proposed reactive cost function is determined by following two stages:

Stage1: The generator operates on its capability curve.
In this situation when the generator produces the maximum active power ($P_{\max}$), its production cost equals to cost ($P_{\max}$) and no reactive power is produced. To generate reactive power in amount of Q_i (Fig. 1), which corresponds to its nominal rating with unity power factor, the active power production should be reduced to P_i such that:

$$P_i = \sqrt{P_{\max}^2 - Q_i^2}$$
$$\Delta P = P_{\max} - P_i \qquad (4)$$

where, ΔP represents the amount of reduction in active power in order to produce reactive power. Therefore, to estimate the cost of reactive power Q_i, we should calculate all costs imposed on generator as follows:

$\cos t(P_{\max})$: cost of producing active power equal to $P_{\max}$ in one hour.

$\cos t(P_{\max} - \Delta P)$: cost of producing both active and reactive power with the amounts P_i and Q_i, respectively.

$\cos t(P_{\max}) - \cos t(P_{\max} - \Delta P)$: cost reduction due to reducing active power generation ΔP for producing reactive power Q_i. This represents the cost of reactive power production when the operating point of generator is moved from point 1 to point 2 on capability curve (see Fig. 1), yielding

$$\cos t(P_{\max}) - \cos t(P_{\max} - \Delta P) = \cos t(Q_i) + \frac{\Delta P}{P_{\max}} \cos t(P_{\max})$$

where, $\frac{\Delta P}{P_{\max}} \cos t(P_{\max})$ is related to the change of operating point (In fact this represents the cost of energy related to ΔP MW in one hour when the generator is operated at its nominal rating).
Therefore, from the above equation it can be concluded that:

$$\cos t(Q_i) = \cos t(P_{\max}) - \cos t(P_{\max} - P_i) - \frac{\Delta P}{P_{\max}} \cos t(P_{\max})$$
$$\cos t(Q_i) = \frac{P_{\max} - \Delta P}{P_{\max}} \cos t(P_{\max}) - \cos t(P_{\max} - P_i) \qquad (5)$$

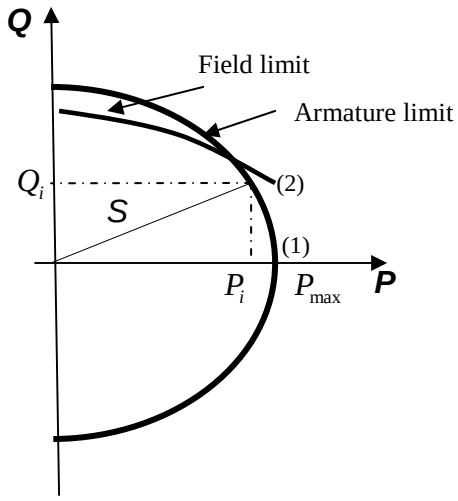

Fig. 1. Capability curve of generator

Now, we should express the $\cos t(Q)$ as a function of Q. By assuming that the full potential of the generator capability is used, we may conclude that its operating point will always be such that its current will be equal to its nominal value and we will be able to write Q as a function of P (equation 5). Therefore, by considering Q as variable and using equations 4 and 5, its production cost can be calculated for different values of Q. The results, interpolated by Newton-Gregory polynomial, confirm that they can accurately be fitted into a quadratic polynomial form as below:

$$\cos t(Q) = a_q Q^2 + b_q Q + c_q \qquad (6)$$

Stage2: The generator operates inside the capability curve.
In this condition, the generator does not necessarily need to reduce its active power, for producing the reactive power. For example, once a generator

produces $0.9P_{\max}$, it can produce the reactive power without reducing its active power until the armature current limitation is reached. Then, the cost of reactive power is determined as bellow:

$$P_i = \sqrt{(0.9P_{\max})^2 - Q_i^2}$$

$$\Delta P = 0.9P_{\max} - P_i$$

$$\cos t(Q_i) = \frac{0.9P_{\max} - \Delta P}{0.9P_{\max}} \cos t(0.9P_{\max}) - \cos t(0.9P_{\max} - P_i) \qquad (7)$$

Now, considering Q as a variable, the quadratic function of reactive power production cost is determined in such operating points. Considering the fact that the active power production of generator is generally not less than $0.7P_{\max}$, obtaining the reactive cost curve is stopped at this level. And then, the envelope of the provided cost curves is treated as the reactive cost curve for the present stage.

This equation (7) is very simple and as it is extracted from the power cost function of the generator, it is more realistic and can provide accurate results in reactive power pricing as compared with conventional empirical approximate method. The proposed cost function, as compared with previously used methods, not only considers the operational cost imposed to the system due to reactive power support, but also the opportunity cost is taken into account. Furthermore, investment cost in this equation is accurately included. Fig. 2 shows the plotted cost curves for active power and the proposed reactive power formulation. From this figure, it can be observed that both cost curves show similar characteristics. However, as it should be, the cost of reactive power is much smaller than that of active power.

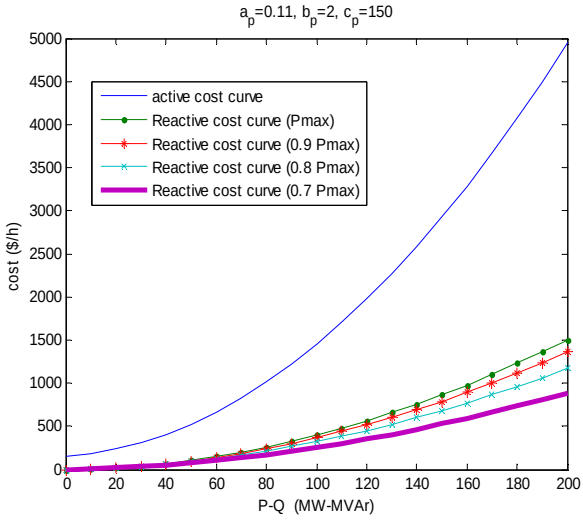

Fig. 2. Active cost curve and proposed reactive cost curve

In figures 3 and 4, the cost allocated to reactive power, obtained by using the proposed method is compared with those obtained from conventional and the triangular methods for two different generators having the following parameters:

Gen1: $a_P = 0.085, \quad b_P = 6.5, \quad c_P = 200$

Gen2: $a_P = 0.11, \quad b_P = 2, \quad c_P = 150$

While in both cases, the triangular method is almost compatible with our proposed method; it can be easily observed that the conventional cost method may not be reasonable (Fig. 4). This is mainly due to the fact that the investment cost is not included in the pricing of reactive power in the conventional method. Therefore, depending on the values of a_p , b_p and c_p , the results obtained by the conventional method may differ significantly from the actual cost of reactive power production imposed on generator.

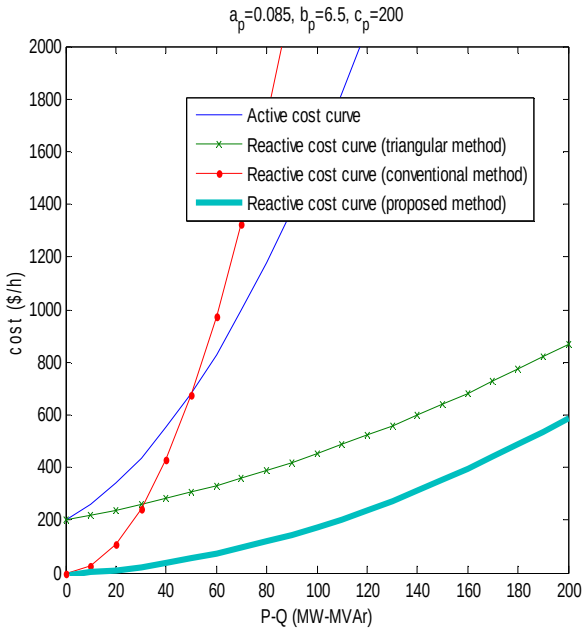

Fig. 3. Comparison of the new method with conventional and triangular methods for generator 1

3. Reactive Power Pricing

Active and reactive marginal prices are normally obtained through solving the optimal power flow in which an objective function subject to a set of equality and inequality constraints is minimized. In this paper, we also propose a new frame for reactive power cost allocation which covers all costs associated with reactive power generation in objective function of optimization problem.

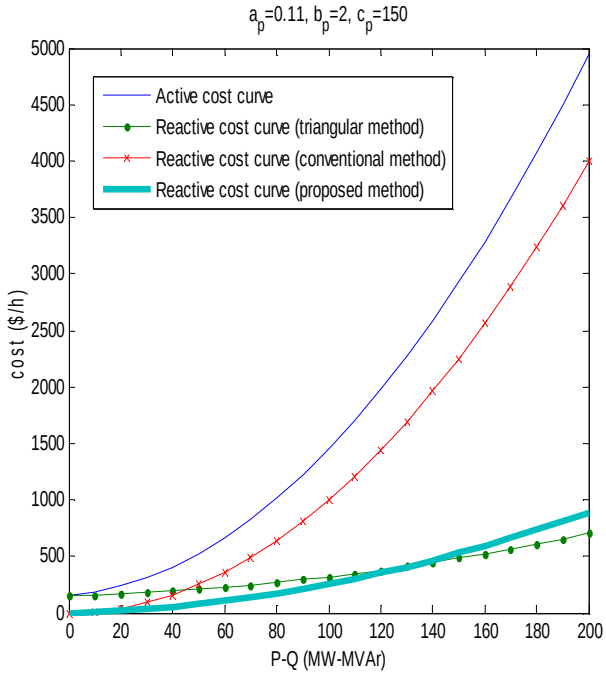

Fig. 4. Comparison of the new method with conventional and triangular methods for generator 2

3.1. Objective Function

The total costs imposed on generators and VAr sources including the active and reactive power losses are defined as an objective function in our new formulation. To accurately include the effect of marginal cost of different generators on the total costs imposed by losses, we should first evaluate the amount of active and reactive power losses attributed to each generator. This is achieved by using a well known tracing algorithm. As a result, the cost of losses assigned to each generator, can be calculated fairly. To do this, we have formulated the objective function as the summation of cost functions for pure consumed active and reactive power as well as the cost functions for losses. Therefore, we can write the objective function as below:

$$C_{total} = \sum_{i=1}^{N_g} [Cost(P_{G_i} - \Delta P_{G_i}) + Cost(Q_{G_i} - \Delta Q_{G_h}) + \Delta P_{G_i}.\lambda_{P_i} + \Delta Q_{G_i}.\lambda_{Q_i}] + \sum_{j=1}^{N_C} C_{cj}(Q_{cj}) \quad (8)$$

where,

N_g : Number of generators

N_c : Number of buses in which the capacitor banks are installed

$Cost(P_{G_i})$: Active power cost function of generator i

$Cost(Q_{G_i})$: Reactive power cost function of generator i

$C_{cj}(Q_{cj})$: Capital cost function of capacitor bank in j^{th} bus

λ_{P_i} : Active power price in generator i

λ_{Q_i} : Reactive power price in generator i

ΔP_{G_i} : Active power loss allocated to generator i

ΔQ_{G_i} : Reactive power loss allocated to generator i

$P_{G_i} - \Delta P_{G_i}$: Active power production by generator i without considering active loss

$Q_{G_i} - \Delta Q_{G_i}$: Reactive power production by generator i without considering reactive loss

$\Delta P_{G_i}, \Delta Q_{G_i}$ are calculated using a tracing based loss allocation algorithm [18].

The charge of capacitors is assumed to be proportional to the amount of the reactive power output purchased and can be expressed as:

$$C_{cj}(Q_{cj}) = r_j Q_{cj} \quad (9)$$

where,

r_j : Production cost at location j

Q_{cj} : Amount purchased at location j

The production cost of capacitor is assumed as its capital investment return, which can be expressed by equation (10) as its depreciation rate.

$$r_j = \frac{investment \ \ \cos t}{operating \ \ hours} \quad (10)$$

For example, if the investment cost of a capacitor is 11600 $/MVA and its average working rate is 2/3 and life span is 15 years, the cost or depreciation rate of capacitor can be calculated by:

$$r_j = \frac{\$11600}{15 \times 365 \times 24 \times 2/3} = 0.1324 \ \$/MVAh$$

In the proposed approach, both active and reactive losses allocated to each generator are included in the objective function. Therefore, it guarantees a more accurate and non-discriminative pricing scheme for active and reactive power.

3.2. Constraints

The constraints for the problem are the standard set of equality and inequality constraints normally considered in OPF. In fact, the set of equality constraints represent the standard power flow equations for active and reactive power and the set of inequality constraints represent the physical and security limits of the system as below.

Load flow equations:

$$\sum_{i=1}^{N_g} P_{G_i} - \sum_{i=1}^{N} P_{D_i} - P_{loss} = 0$$

$$\sum_{i=1}^{N_g} Q_{G_i} - \sum_{i=1}^{N} Q_{D_i} - Q_{loss} = 0$$

where,

$$P_{loss} = \sum_{i=1}^{N} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_{loss} = \sum_{i=1}^{N} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} + \delta_j - \delta_i)$$

Active and reactive power generation limits:

$$P_{G_{\min}} \le P_{G_i} \le P_{G_{\max}}$$

$$Q_{G_{\min}} \le Q_{G_i} \le Q_{G_{\max}}$$

$$P_{G_i}^{\ 2} + Q_{G_i}^{\ 2} \le S_{G_i,\max} \qquad i = 1,\ldots,N_g$$

Capacitor reactive power generation limits:

$$0 \le Q_{cj} \le Q_{cj,\max} \qquad j = 1,\ldots,N_C$$

Transmission line limits:

$$P_{ij} \le P_{ij,\max}$$

Bus voltage limits:

$$|V_i|_{\min} \le |V_i| \le |V_i|_{\max} \qquad i = 1,\ldots,N \qquad (11)$$

In the above formulas we have:

N: number of buses of the network

P_{G_i}, Q_{G_i}: Supply of active and reactive power in i^{th} bus

P_{D_i}, Q_{D_i}: Active and reactive demand in i^{th} bus

$S_{G_i,\max}$: Maximum apparent power in bus i

$Q_{cj,\max}$: Maximum reactive power output of the capacitor

$V_i = |V_i| \angle \delta_i$: Voltage phasor in bus i

$Y_{ij} \angle \theta_{ij}$: ij^{th} Element of admittance matrix

4. Case Study

To investigate the validity of the proposed algorithm, it has been applied to IEEE 9 and 30 bus systems with a typical daily load as shown in Fig. 5. The 9-bus test system has 3 generator buses and a capacitor bank installed in bus 6. Tables 1 and 2 show the parameters of these systems.

To be able to make an analytical comparison between the proposed method and the previous algorithms, two different scenarios have been analyzed [19]. In the first scenario, the network losses and its effect on the cost function are neglected while in the second approach not only the network losses are taken into account but the cost of the local capacitor banks is also included in the objective function.

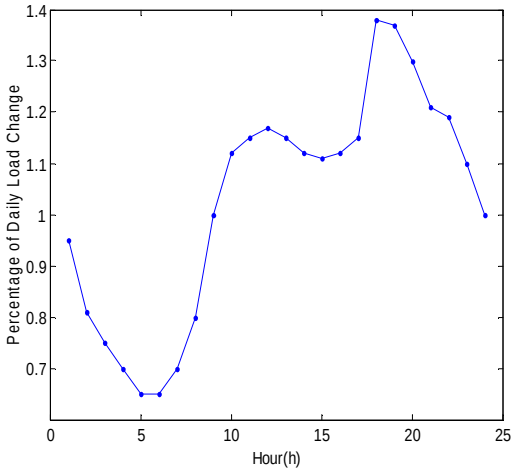

Fig. 5. Percentage of load daily change

Table.1. Generators characteristics

No. bus	a_p	b_p	c_p	P_{ma} MW	P_{mi} MW	Q_{ma} MVAr	Q_{mi} MVAr
1	.11	5	150	250	10	300	-300
2	.08	1.2	600	300	10	300	-300
3	.12	1	335	270	10	300	-300

Table. 2. Load characteristics

No. bus	Active power MW	Reactive power MVAr
5	90	30
7	100	35
9	125	50

Scenario No. 1:

For this scenario, we have simulated three different cases as below:

1) In case 1, only the cost of active power produced by generators is considered in the objective function.
2) In case 2, the costs for both active and reactive power are considered in the objective function. In this case, the cost function has been modeled based on the conventional reactive cost formulation.
3) In case 3, while the costs for both active and reactive power are included in the objective function, the cost function for reactive power has been modeled according to our proposed formulation.

Table 3 shows simulation results for three above mentioned cases. The active and reactive marginal prices during 24 hours of the typical day for generator 1 are shown in Fig.7.

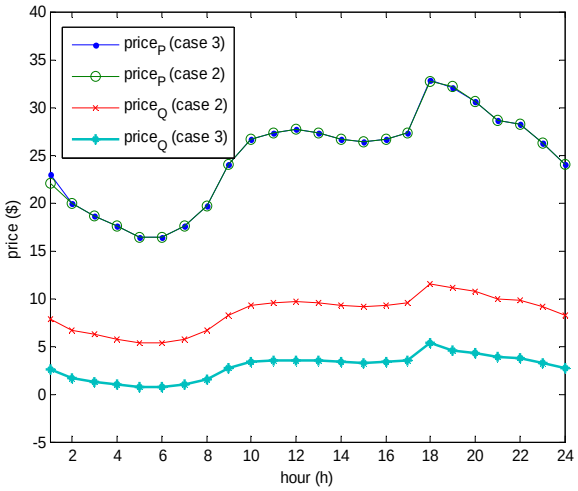

Fig.7. Active and reactive marginal prices in conventional and proposed methods

Comparing the results for these three cases, it can be easily concluded that:

1) Irrespective of the cost function modeled for reactive power, its cost and consequently its price is much lower than that of the active power. However, due to the fact that reactive power is very important for enhancement of secure system operation, it can not be ignored.
2) As our proposed method for reactive power cost allocation is based on a more accurate modeling in compare with the approximate conventional methods, it can be easily observed that the cost allocated for reactive power in our formulation may be significantly different from that of conventional methods. It should be emphasized that this differences arise from that fact that in the conventional models the investment and opportunity cost components are not considered.
3) As it can be seen, the cost dedicated to reactive power in our model is much greater than that of conventional ones, which in turn, may imply a positive signal for investors to think about investment on reactive power supplies.

Scenario No. 2:

In this scenario, we have emphasized on the analysis and effects of allocating losses to all generators of the network using the tracing algorithm. In this approach, at first the portion of losses produced by each generator is determined based on tracing algorithm and then its accurate cost is evaluated using our proposed formulation given by (8).

For this scenario, we have simulated two different cases as below:

1) In case 1, the network losses and their effects on the cost function have been modeled according to proposed formulation.
2) In case 2, the reactive power production costs of capacitor banks are also considered.

The results obtained from IEEE 9-bus system are shown in Table 4.

The results confirm that:

1) The total costs for both cases of scenario No. 2 are smaller than that of case 3 in scenario No. 1. (Total cost for cases 1 and 2 in scenario No. 1 are smaller than the cost in other cases. However, it should be reminded that this is just due to the fact that the cost is not valued accurately.)
2) When all of reactive power production costs are taken into consideration, the corresponding reactive power prices increases.
3) As our tracing based proposed method, allocates the active and reactive losses to different generators and their costs are evaluated accurately, it is more compatible with the open access networks. Therefore, it will not lead to unfair and wrong signals to generators.

In order to show the performance of the proposed method it is also applied to IEEE 30-bus system for base load.

Table.3. Analysis results for different cases of scenario 1

	No. bus	P_g (MW)	Q_g (MVAr)	λ_p (\$/MW)	λ_Q (\$/MVAr)	$\cos t_Q$ (\$)	$Cost_{total}$ (\$)
Case1	1	113.67	36.36	30.026	--	--	6.273155E+3
	2	82.63	52.69	30.026	--	--	
	3	118.45	30.85	30.026	--	--	
Case2	1	113.76	16.51	30.027	8.256	68.218	6.746710E+3
	2	82.67	15.80	30.003	8.221	64.986	
	3	118.51	82.66	30.036	8.266	341.68	
Case3	1	113.76	42.23	30.027	2.369	67.455	6.47559E+3
	2	82.75	31.96	30.027	2.775	59.081	
	3	118.47	40.79	30.027	3.0002	75.904	

Table.4. Analysis results for different cases of scenario 2

	No. bus	λ_p (\$/MW)	λ_Q (\$/MVar)	$Cost_{total}$ (\$)
Case1	1	30.025	2.361	6.41439E+3
	2	30.038	2.765	
	3	30.036	3.0002	
Case2	1	30.0254	2.53	6.43763E+3
	2	30.0373	2.93	
	3	30.0348	3.25	
	6	30.0298	3.278	

Table 5 provides the parameters of this system. The simulation results for base load case study are shown in Table 6. For the purpose of comparison, the results of the conventional method are shown in the same table. It should be noted that when the reactive power produced by generator is negative, the reactive marginal price is set to zero.

Table. 5. Generators characteristics for IEEE 30-bus system

No. Gen. bus	a_p	b_p	c_p
1	.02	2	100
2	.0175	1.75	250
13	.0625	1	150
22	.0835	1.25	200
23	.025	3	230
27	.025	3	180

It can be seen that the reactive marginal prices in our model are greater than that of the conventional method, so it provides a positive signal to investors leading to more investment on reactive power supplies. This will result in a more secure operation of the system in the future specially in restructured power systems. It should be emphasized that in spite of the fact that reactive power is very important for enhancement of secure operation of the system, its cost is not compared with that of the active power.

5. Conclusion

In this paper a new method for reactive power pricing has been proposed. The proposed method utilizes the accurate relation between active and reactive power to assign an accurate quadratic function for cost function of reactive power support. Using optimization techniques, active and reactive losses allocated to each generator are considered utilizing the tracing algorithm in reactive power pricing. The results confirm that the reactive power cost allocation techniques, which are based on approximate conventional methods, may result in wrong signals for market participants. This, in turn, may result in threatening the secure operation of the system as well. However, such drawbacks are improved in our proposed method. The proposed method is simple, flexible and more accurate in compare with conventional methods. Therefore, it is more compatible with open access deregulated systems.

Table.6. Analysis results for 30 bus system

	No. bus	P_g *(MW)*	Q_g *(MVAr)*	λ_p *(\$/MW)*	λ_Q *(\$/MVAr)*	$Cost_{total}$ *(\$)*
Conventional method	1	43.29	9.53	3.731	0.962	1.64612E+3
	2	56.49	3.85	3.727	0.754	
	13	16.91	-16.02	3.114	0.00	
	22	39.41	9.85	3.873	0.952	
	23	11.59	-10.94	3.579	0.00	
	27	13.03	13.04	3.6519	1.023	
Proposed method	1	48.75	11.23	3.85	1.12	1.73341E+3
	2	56.72	5.362	3.75	0.93	
	13	18.94	-20.23	3.352	0.00	
	22	40.14	12.28	3.901	1.26	
	23	12.5	-9.35	3.62	0.00	
	27	14.25	11.65	3.712	1.11	

References

[1] Reactive power support services in electricity markets. Power System Engineering Research Center (PSERC) publication 2001.

[2] F. Alvarado, "Reactive power as an identifiable ancillary service", Prepared for Alberta, Ltd.2003.

[3] T. Niknam, et al., "Reactive power pricing in deregulated environments using novel search methods", IEEE proceeding of third international conference on machine learning and Cybemetics 2004, Shanghai 26-29, pp. 4234-4240.

[4] Y. Zhao, et al., "A cost allocation and pricing method for reactive power services in the new deregulated electricity market environment", IEEE transmission and distribution conference, Asia and Pacific, Dalian, China 2005, pp. 1-6.

[5] XJ. Lin, et al., "Pricing of reactive support ancillary services", IEE proceeding generation transmission distribution 2005, 152, pp. 616-622.

[6] M. Muchayi, et al., "A summary of algorithm in reactive power pricing", Electrical Power and Energy System 21, 1999, pp. 119-125.

[7] A. Katebi, et al., "Application of the ant colony search algorithm to reactive power pricing in an open electricity market", American journal of applied sciences 6, Vol. 5, 2009, pp. 956-963.

[8] W. Chu, et al., "Allocating the costs of reactive power purchased in an ancillary services market by modified Y-bus matrix method", IEEE transaction on power systems 2004, 9, 1, pp. 174-180.

[9] MJ. Rider, et al., "Application of a nonlinear reactive power pricing model for competitive electric markets", IEE proceeding generation transmission distribution 2004, 151, 3, pp. 407-415.

[10] S. Kotsan, "Efficient pricing of a bundled product of both real and reactive power", IEEE power system conference and exposition 10-13, 2004, l, pp. 108-123.

[11] K. Xie, et al., "Calculation and decomposition of spot price using interior point nonlinear optimization methods", Electrical Power and Energy System26, 2004, pp. 379-388.

[12] S. G. Seifossadat et al., "Reactive power pricing in competitive electric markets using a sequential linear programming with considered investment cost of capacitor bank", International journal of innovations in energy systems and power, Vol. 4, No. 1, 2009, pp. 29-43.

[13] CY. Chung, et al., "Cost-based reactive power pricing with voltage security consideration in restructured power systems", Electric Power Systems Research 70, 2004, pp. 85-92.

[14] L. Singh, J. Dhillon, "Secure multi objective real and reactive power allocation of thermal units", Electric Power and Energy Systems 30, 2008, pp. 594-602.

[15] R. Deksnys, R. Staniulis, "Pricing of reactive power services", Oil shale, Estonian academic publish, 2007, 24, pp. 363-376.

[16] A. Tiawari, V. Ajjarapu, "Reactive power cost allocation based on modified power flow tracing methodology", IEEE Power engineering society general meeting 24-28, 2007, pp. 1-7.

[17] K. Ro, "Calculation of reactive power charges under competition of electric power industries", Springer, Electrical Engineering, 2003, 85, pp. 169-175.

[18] JW. Bialak, et al., "Proportional sharing assumption in tracing methodology", IEE proceeding generation transmission distribution 2004, 151, 4, pp. 526-532.

[۱۹] مصطفی رجبی مشهدی، محمد حسین جاویدی، محمد صادق قاضی‌زاده،"برنامه‌ریزی همزمان انرژی و ذخیره کنترل فرکانس اولیه با لحاظ کردن قابلیت‌های فنی واحدهای نیروگاهی پیشرفته" مجله انجمن مهندسین برق و الکترونیک ایران، سال ششم، شماره دوم، پاییز و زمستان ۸۸