

Power System Analysis for Nonsinusoidal Steady State Studies Based on Wavelets

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Abstract:

In this paper power system model is represented in a new domain that relates to Multi-Resolution Analysis (MRA) space. By developing mathematical model of elements in this space using Galerkin method, a new alternative method for power system simulation in nonsinusoidal and periodic conditions is developed. The mathematical formulation and characteristics of new proposed space is expressed. Also the relation between this domain and spectral analysis is presented. Transmission line and switching devices modeling in the proposed domain is investigated. To consider frequency dependency of line parameters and to obtain harmonics information, the relationship between this dependency and the new suggested domain is discussed. Then the new algorithm is presented and demonstrated for two case studies.

Keywords: Power system model, Multiresolution Analysis, Galerkin method, Spectral analysis, Distributed transmission line.

Submission date: Oct. 24, 2007

Acceptance date: Feb. 8, 2009

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1. Introduction

New power systems include nonlinear, switching and frequency dependent elements. For example overhead lines, underground cables and impedances of transformers are frequency dependent elements due to skin effect and existence of harmonics that produced with switching devices. To investigate and estimate the harmonic content and propagation in a network with such elements, an algorithm for calculating the periodic steady state solution is required. Different algorithms to this end have been developed by many researchers. These algorithms are classified in terms of their formulation methodologies, into three categories: harmonic domain methods [1, 2], time domain methods [3, 4] and hybrid methods [5, 6]. In the harmonic domain method all network elements are expressed in the harmonic domain. Noninteger harmonics considered in harmonic sources is however an important concern in this method. Also, when one deals with the devices such as switching and nonlinear elements, the number of considered harmonics and coupling between them can make difficulties in numerical solutions [1].

In time domain method the nonlinearity and switching are modeled with ease, but frequency dependency of elements is a complicated concern. In the other hand, as in harmonic studies for the case of the steady state solution, the transient response of system must be eliminated by adjusting the initial conditions [4].

In recent years wavelet transform is used in power engineering. Despite the major abilities of this tool, its application is limited to signal processing for power quality usage. The base of this transform refers to MRA space, and this space has unique features. By using differential equations theory, the power system such as Fourier space can be represented in this space. New methods for power system analysis in MRA space are introduced by other authors in [7, 8]. The speeds of solution described in these papers make them inefficient for numerical simulation and are hence impractical for a real power system with relevant large dimension.

In this paper MRA space is used for power system simulation in nonsinusoidal and periodic conditions. Wavelet-Galerkin method guarantees the validation of this study from the mathematical point of view [13]. In MRA space a new space suitable for numerical analysis of power system is used. Investigating the features of the new space shows close similarity between it and the harmonic domain.

The paper is organized as follows: In section II a brief description of mathematical theory is presented. In section III the mathematical formulation is expressed. In this section the representation of power system in the new suggested domain is described. In section IV the relationship between this domain and spectral analysis is illustrated. In section VI distributed transmission line modeling is investigated Two case studies in sections V and VII are simulated in the new domain and the results of these simulations are compared with a time domain

simulation.

2. Mathematical Theory

2.1. Galerkin Method

The Galerkin method is one of the most reliable methods for finding numerical solution to differential equations [13]. Its simplicities make it perfect for many applications. The Galerkin approach is based on finding a functional basis for the solution space of the equation, then projecting the solution on it, and minimizing the residual with respect to the functional basis. Standard polynomial basis or trigonometric basis is used for Galerkin method. However wavelets used to describe MRA space often offers additional improvement to the above by providing both time and frequency localization. This means not only all dilations from an unconditional orthonormal bases of $L^2(\mathfrak{R})$ are translated but scaling functions of all dilations for such bases for $V_j \subset L^2(\mathfrak{R})$ are also translated.

2.2. Multiresolution Analysis

In this section the orthonormal basis of compactly supported wavelets is reviewed briefly. The orthonormal basis of compactly supported wavelets of $L^2(\mathfrak{R})$ is formed by the dilation and translation of single function $\psi(x)$ [10]:

$$\psi(x) = 2^{-j/2} \psi(2^{-j}x - k) \quad (1)$$

Where $j, k \in Z$, the function $\psi(x)$ has a companion: the scaling function $\phi(x)$, and these functions satisfy the following relations:

$$\phi(x) = \sqrt{2} \sum_{k=0}^{J-1} h_k \phi(2x - k) \quad (2)$$

$$\psi(x) = \sqrt{2} \sum_{k=0}^{J-1} g_k \phi(2x - k) \quad (3)$$

The number J of coefficients in (2) and (3) is related to the number of vanishing moments M , and for the wavelets here $J = M$. The wavelet basis induces a MRA on $L^2(\mathfrak{R})$, i.e. the decomposition of Hilbert space into a chain of closed spaces:

$$\dots \subset V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \subset \dots \quad (4)$$

Such that:

$$\bigcap_{j \in Z} V_j = \{0\}, \quad \bigcup_{j \in Z} V_j = L^2(\mathfrak{R}) \quad (5)$$

By defining W_j as an orthonormal complement of V_j in V_{j+1} :

$$V_{j+1} = V_j \oplus W_j \quad (6)$$

The space $L^2(\mathfrak{R})$ is represented as a direct sum:

$$L^2(\square) = \bigoplus_{j \in \mathbb{Z}} W_j \quad (7)$$

The coefficients $H = \{h_k\}_{k=0}^{k=J-1}$ and $L = \{g_k\}_{k=0}^{k=J-1}$ in (2) and (3) are quadrature mirror filters. With a MRA, one can use $\varphi_{n,k}(x)$ and $\psi_{n,k}(x)$ as the basis functions for Galerkin method.

2.3. Wavelet-Galerkin Solution of a Periodic Problem

To have a view of solution method in the MRA space, consider the following problem:

$$L(u) = f, \quad f(0) = f(T) \quad (8)$$

Where L is the differentiation operator. Suppose u and f have projections to MRA space such as below:

$$u_{j+1}(x) = \sum_{k=0}^{2^j-1} v_{k,j}^u \varphi_{j,k}(x) + \sum_{k=0}^{2^j-1} w_{k,j}^u \psi_{j,k}(x) \quad (9)$$

$$f_{j+1}(x) = \sum_{k=0}^{2^j-1} v_{k,j}^f \varphi_{j,k}(x) + \sum_{k=0}^{2^j-1} w_{k,j}^f \psi_{j,k}(x) \quad (10)$$

Using (9) and (10), equation (8) can be written as follow:

$$\sum_{k=0}^{2^j-1} v_{k,j}^u L(\varphi_{j,k}(x)) + \sum_{k=0}^{2^j-1} w_{k,j}^u L(\psi_{j,k}(x)) = \sum_{k=0}^{2^j-1} v_{k,j}^f \varphi_{j,k}(x) + \sum_{k=0}^{2^j-1} w_{k,j}^f \psi_{j,k}(x) \quad (11)$$

By taking an inner product with φ and ψ , in the MRA space the equation (8) can be written in this matrix form [15]:

$$S_j u = f \quad (12)$$

The decomposition $V_{j+1} = V_j \oplus W_j$ allows the operator S_j to be spitted into four pieces (W_j is called the wavelet space and the detail or fine-scale component of V_{j+1}) which can be written as follows:

$$\begin{pmatrix} A_{sj} & B_{sj} \\ C_{sj} & T_{sj} \end{pmatrix} \begin{pmatrix} d_{xj} \\ s_{xj} \end{pmatrix} = \begin{pmatrix} d_{ff} \\ s_{ff} \end{pmatrix} \quad (13)$$

Where:

$$\begin{aligned} A_{sj} : W_j &\rightarrow W_j, & B_{sj} : V_j &\rightarrow W_j \\ C_{sj} : W_j &\rightarrow V_j, & T_{sj} : V_j &\rightarrow V_j \end{aligned} \quad (14)$$

And $d_{xj}, d_{ff} \in W_j, s_{xj}, s_{ff} \in V_j$ are the L^2 -orthonormal projections of x and f onto W_j and V_j spaces. The projection s_{xj} is the coarse-scale component of the solution x , and d_{xj} is the fine-scale component. To solve (12):

$$R_{sj} = T_{sj} - C_{sj} A_{sj}^{-1} B_{sj} \quad (15)$$

$$s_x = R_{sj}^{-1} (s_f - C_{sj} A_{sj}^{-1} d_f) \quad (16)$$

$$d_x = A_{sj}^{-1} (d_f - B_{sj} s_x) \quad (17)$$

At this stage T_{sj} is selected and investigated. As the problem described above is periodic and supposing that the operator L is equal to d^m/dx^m , the general form of T_{sj} is:

$$T_{sj} = \begin{pmatrix} \Omega_0^{(m)} & \dots & \Omega_{-N+2}^{(m)} & \dots & \Omega_{-N-2}^{(m)} & \dots & \Omega_1^{(m)} \\ \Omega_1^{(m)} & \Omega_0^{(m)} & \dots & \Omega_{-N+3}^{(m)} & \dots & 0 & \dots & \Omega_2^{(m)} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots & \vdots & \vdots \\ \Omega_{-1}^{(m)} & \Omega_{-2}^{(m)} & \dots & 0 & \dots & \Omega_{N-3}^{(m)} & \dots & \Omega_0^{(m)} \end{pmatrix} \quad (18)$$

The general forms of the other pieces of S_j are also similar to T_{sj} . For a circulant matrix [16] such as T_{sj} , the eigenvalues λ_α are:

$$\lambda_\alpha = \sum_{k=-N+2}^{N-2} \Omega_k^{(m)} \exp(-2\pi i \alpha k / n), \quad \alpha = 0, 1, \dots, n-1 \quad (19)$$

And the corresponding orthonormal eigenvectors v_α are:

$$(v_\alpha)_k = \frac{(-1)^\alpha}{\sqrt{n}} \exp(-2\pi i \alpha k / n), \quad k = 0, 1, \dots, n-1 \quad (20)$$

These relations lead to provision of quasi-diagonal form of represented operators in MRA space without using conventional methods in a lower time. Using diagonal form offers several advantages that are explained in the next parts. Using (19) and (20), (13) can be rewritten as:

$$\begin{pmatrix} \overline{A_{sj}} & \overline{B_{sj}} \\ \overline{C_{sj}} & \overline{T_{sj}} \end{pmatrix} \begin{pmatrix} \hat{d}_{xj} \\ \hat{s}_{xj} \end{pmatrix} = \begin{pmatrix} \hat{d}_{ff} \\ \hat{s}_{ff} \end{pmatrix} \quad (21)$$

Where in (21):

$$\hat{d}_{xj} = \Gamma^{-1} d_{xj}, \quad \hat{s}_{xj} = \Gamma^{-1} s_{xj} \quad (22)$$

$$\hat{d}_{ff} = \Gamma^{-1} d_{ff}, \quad \hat{s}_{ff} = \Gamma^{-1} s_{ff} \quad (23)$$

In these equations Γ is the modal matrix. The columns of Γ are calculated using (20). $\overline{A_{sj}}, \overline{B_{sj}}, \overline{C_{sj}}$ and $\overline{T_{sj}}$ are diagonal matrices and their elements calculated by (19).

So to calculate $\hat{d}_{xj,i}$ and $\hat{s}_{xj,i}$ (the i^{th} values of \hat{d}_{xj} and \hat{s}_{xj}), the following equation must be solved:

$$\begin{pmatrix} a_{sj}^i & b_{sj}^i \\ c_{sj}^i & t_{sj}^i \end{pmatrix} \begin{pmatrix} \hat{d}_{xj,i} \\ \hat{s}_{xj,i} \end{pmatrix} = \begin{pmatrix} \hat{d}_{jj,i} \\ \hat{s}_{jj,i} \end{pmatrix} \quad (24)$$

The volume of calculations is decreases using the above technique significantly. In the other word instead of calculating the inverse of matrices with $N/2 \times N/2$ dimensions in (12), (24) is used for $N/2$ iterations. For problems with small dimensions this method seems not to be beneficial, however as shown in the following sections in this article this approach could be very useful for solving differential equations of large power systems, as in such systems the dimension of S_j in (12) is obtained by the multiplication of the system dimension and the number of considered samples (N). Multiplications with Γ introduces a new mapping to a new domain.

3. Power System Representation in The New Domain

3.1. Linear Elements Representation

The aim of this part is to obtain the expression for linear elements using mathematical operator representation in the new suggested domain. In this work modeling in the MRA space has been carried out on the same basis as suggested by other researchers [7, 8]. This mathematical expression should be a complete and meaningful one for power system studies in nonsinusoidal and periodic conditions. Major linear elements for power system, i.e. Resistors, Inductors and Capacitors are discussed briefly underneath:

3.1.1. Resistor

The relationship between voltage and current of a resistor in the time domain is:

$$v(t) = ri(t) \quad (25)$$

(25) in the new domain (as explained in section 2) is:

$$\begin{pmatrix} V_{dj} \\ V_{sj} \end{pmatrix} = r \begin{pmatrix} U & 0 \\ 0 & U \end{pmatrix} \begin{pmatrix} I_{dj} \\ I_{sj} \end{pmatrix} \quad (26)$$

In (26), U is a unit matrix with dimensions of $N/2^{(j_{max}-j+1)}$.

3.1.2. Inductor

The relationship between voltage and current of an inductor is:

$$v(t) = \ell \frac{di(t)}{dt} \quad (27)$$

So N point discretization of (27) leads to:

$$\underline{V} = \ell [D_T] \underline{I} \quad (28)$$

In (28) D_T is the discretized form of derivative operator and ℓ is a scalar. As the purpose is to obtain periodic solution, D_T is a circulant matrix. In the MRA space (28) can be written as:

$$\begin{pmatrix} V_{dj} \\ V_{sj} \end{pmatrix} = \ell \cdot W D_T \begin{pmatrix} I_{dj} \\ I_{sj} \end{pmatrix} \quad (29)$$

Where:

$$W D_T = \begin{pmatrix} H D_T H & H D_T L \\ L D_T H & L D_T L \end{pmatrix} = \begin{pmatrix} A_{sj} & B_{sj} \\ C_{sj} & T_{sj} \end{pmatrix} \quad (30)$$

To transfer (29) from highest level (finest scale) to next lower level (coarser scale) and respectively in a hierarchical form to other levels (scales) of MRA space, D_T is substituted with $L D_T L$ of the higher resolution level. Of course in each subsequent level the dimensions of matrices will be different from previous ones and its magnitude is divided by 2. The sub-matrices of $W D_T$ have a circulant form and this feature is specific to all of orders of derivative operator in MRA space. Also the Γ matrix and eigenvectors are the same for all orders in each level of MRA space. Rewriting (30) using (19) and (20) leads to obtain a quasi-diagonal form such as follow:

$$\begin{pmatrix} \hat{V}_{dj} \\ \hat{V}_{sj} \end{pmatrix} = \ell \cdot \overline{W D_T} \begin{pmatrix} \hat{I}_{dj} \\ \hat{I}_{sj} \end{pmatrix}, \quad \overline{W D_T} = \begin{pmatrix} \overline{HH} & \overline{HL} \\ \overline{LH} & \overline{LL} \end{pmatrix} \quad (31)$$

Where $\ell \cdot \overline{W D_T}$ defines the impedance of inductor in the new suggested domain. There are four sub-matrices for impedance definition of inductor, the first sub-matrix (\overline{HH} deals with $W_j \rightarrow W_j$) belongs to high frequency part of level j . Also the fourth sub-matrix (\overline{LL} relates to $V_j \rightarrow V_j$) represents the impedance in low frequency part. The Fig. 1 shows the impedance of an inductor in the new domain for three scales.

By increasing the number of considered scales the accuracy of represented impedance in the new suggested domain is increased. The number of levels (scales) depends on the order of wavelet filter and the number of samples taken. In power system analysis, the frequency band is limited. Therefore, an acceptable accuracy can be reached by considering 3 or 4 levels. Between high frequency part and low frequency part there is a coupling that is represented by \overline{HL} and \overline{LH} , however these parts are not as important as the cited parts (\overline{HH} and \overline{LL}).

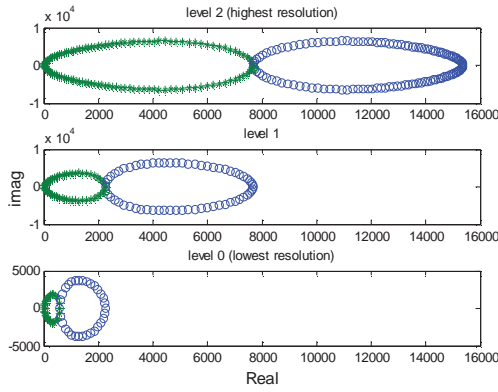


Fig. 1: Impedance of an inductance in the new suggested domain for 3 levels. '*': Low frequency part and, 'o': High frequency part

3.1.3. Capacitor

There is a time domain relationship between the voltage and current of a capacitor which can be discussed as follow:

$$v(t) = \frac{1}{c} \int i(t) dt \quad (32)$$

In discrete form (32) is written as below:

$$\underline{V} = \frac{1}{c} [D_T]^{-1} \underline{I} \quad (33)$$

D_T^{-1} is the discrete form of integral operator in periodic conditions. The transferred form of D_T^{-1} to the new domain is shown by \overline{WD}'_T , this matrix has a structure similar to \overline{WD}_T :

$$\overline{WD}'_T = \begin{pmatrix} \overline{HH}' & \overline{HL}' \\ \overline{LH}' & \overline{LL}' \end{pmatrix} \quad (34)$$

To compute i^{th} value of $\overline{HH}', \overline{HL}', \dots$ this relation is used:

$$\begin{pmatrix} hh'_i & hl'_i \\ lh'_i & ll'_i \end{pmatrix} = \begin{pmatrix} hh_i & hl_i \\ lh_i & ll_i \end{pmatrix}^{-1} \quad (35)$$

where hh_i, hl_i, lh_i and ll_i are the i^{th} values of $\overline{HH}, \overline{HL}, \dots$

3.1.4. Transmission Line

Writing relations of voltage and current drop for a differential part of transmission line leads to:

$$-\frac{\partial v}{\partial x} = ri + \ell \frac{\partial i}{\partial t} \quad (36)$$

$$-\frac{\partial i}{\partial x} = gv + c \frac{\partial v}{\partial t} \quad (37)$$

If (36) replaced in (37), then we have:

$$\frac{\partial^2 v}{\partial x^2} = r_1 \cdot v + r_2 \cdot \frac{\partial v}{\partial t} + r_3 \cdot \frac{\partial^2 v}{\partial t^2} \quad (38)$$

Where:

$$r_1 = r \cdot g, \quad r_2 = r \cdot c + \ell \cdot g, \quad r_3 = \ell \cdot c$$

if (38) is transformed to the new domain for i^{th} element of j^{th} level, one can gets:

$$\begin{pmatrix} \frac{\partial \hat{V}_{dj,i}}{\partial x^2} \\ \frac{\partial \hat{V}_{sj,i}}{\partial x^2} \end{pmatrix} = \begin{pmatrix} r_1 + r_2 hh_i + r_3 hh_i^2 & r_2 hl_i + r_3 hl_i^2 \\ r_2 lh_i + r_3 lh_i^2 & r_1 + r_2 ll_i + r_3 ll_i^2 \end{pmatrix} \begin{pmatrix} V_{dj,i} \\ V_{sj,i} \end{pmatrix} \quad (39)$$

Where hh_i, \dots, ll_i and hh_i^2, \dots, ll_i^2 are the i^{th} diagonal elements of $\overline{HH}, \dots, \overline{LL}$ and $\overline{HH}^2, \dots, \overline{LL}^2$ respectively where:

$$\overline{HH}^2 = H \cdot D_T^2 \cdot H^T, \dots, \overline{LL}^2 = L \cdot D_T^2 \cdot L^T \quad (40)$$

And D_T^2 is the discretized form of second order derivative operator.

As shown in fig.1, adjusting of parameters for a frequency dependent transmission line can be done comfortably.

This approach of modeling can be used for three phase transmission lines using a modal matrix to separate the wave equation of each phase to another. It can be shown the operation of the modal matrix in the new domain is same as time domain.

3.1.5. Switching Devices Modeling

In this part the modeling method for switching devices is investigated briefly. Modeling of these devices is explained by many of researchers for harmonic studies [27]-[31]. Assume that a linear load is connected to network in series with a power electronic switch. The relation between voltage and current of load without considering switch is:

$$i(t) = f(v(t)) \quad (41)$$

Where in (77), f is a linear operator. As the load is in series with the switch, the relation between current and voltage of switching load is:

$$i(t) = p(t) \cdot f(p(t) \cdot v(t)) \quad (42)$$

Where $p(t)$ is switching signal. Switching signal is a periodic function. Discretizing of equation (42) leads to:

$$\underline{I} = [S] \cdot \underline{V} \quad (43)$$

Where:

$$[S] = [P] \cdot [F] \cdot [P] \quad (44)$$

It is not necessary to suppose that the switching device is synchronized with power system frequency. Transferring (44) to the new suggested domain doesn't result a diagonal matrix. This refers to existence of cross-couplings between harmonics. As the transferred matrix is not diagonal, so using of this matrix in network equation directly makes difficulties in numerical solution. To avoid from this problem, this matrix is not considered in admittance matrix directly and the solution is obtained in a repetitive procedure.

3.2. Network Representation

To develop this method for power network simulation, a simple circuit is considered (Fig. 2). Writing the KCL relation leads to:

$$-i_x + \frac{v_{in}}{r} + c \frac{d}{dt} v_{in} + \frac{1}{\ell} \int v_{in} dt = 0 \quad (45)$$

According to the modified nodal method that is used in harmonic analysis:

$$\begin{pmatrix} \frac{1}{r} + cp + \frac{1}{\ell p} & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ i_x \end{pmatrix} = \begin{pmatrix} 0 \\ v_{in} \end{pmatrix} \quad (46)$$

Where p is derivative operator.

Now (37) is discretized as follows:

$$\begin{pmatrix} \frac{1}{r} \cdot U + c \cdot D_T + \frac{1}{\ell} \cdot D_T^{-1} & U \\ U & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ I_x \end{pmatrix} = \begin{pmatrix} 0 \\ V_{in} \end{pmatrix} \quad (47)$$

where U is a unit matrix, response and input vectors are shown in capital letters. If (47) is transformed to MRA space and then diagonal sub-matrices are obtained using (19) and (20) (or mapping (47) to the new domain), the results would be as follows:

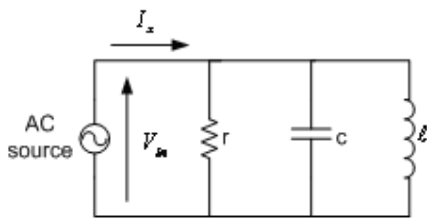


Fig. 2: Parallel RLC circuit

$$\overline{A}_{sj} = \begin{pmatrix} \frac{1}{r} \cdot U + c \cdot \overline{HH} + \frac{1}{\ell} \cdot \overline{HH}' & U \\ U & 0 \end{pmatrix} \quad (48)$$

$$\overline{B}_{sj} = \begin{pmatrix} c \cdot \overline{HL} + \frac{1}{\ell} \cdot \overline{HL}' & 0 \\ 0 & 0 \end{pmatrix} \quad (49)$$

$$\overline{C}_{sj} = \begin{pmatrix} c \cdot \overline{LH} + \frac{1}{\ell} \cdot \overline{LH}' & 0 \\ 0 & 0 \end{pmatrix} \quad (50)$$

$$\overline{T}_{sj} = \begin{pmatrix} \frac{1}{r} \cdot U + c \cdot \overline{LL} + \frac{1}{\ell} \cdot \overline{LL}' & U \\ U & 0 \end{pmatrix} \quad (51)$$

These matrices are written according to (21). As the matrices are mapped to the new suggested domain the input and response (unknown) vectors must be multiplied by Γ^{-1} . Hence:

$$\hat{d}_{fj} = \begin{pmatrix} 0 \\ \Gamma^{-1} \cdot V_{in,dj} \end{pmatrix}, \quad \hat{s}_{fj} = \begin{pmatrix} 0 \\ \Gamma^{-1} \cdot V_{in,sj} \end{pmatrix} \quad (52)$$

$$\hat{d}_{xj} = \begin{pmatrix} \Gamma^{-1} \cdot V_{1,dj} \\ \Gamma^{-1} \cdot I_{in,dj} \end{pmatrix}, \quad \hat{s}_{xj} = \begin{pmatrix} \Gamma^{-1} \cdot V_{1,sj} \\ \Gamma^{-1} \cdot I_{in,sj} \end{pmatrix} \quad (53)$$

Suppose that the purpose is to calculating the i^{th} value of \hat{d}_{xj} and \hat{s}_{xj} . As the sub-matrices of \overline{A}_{sj} , \overline{B}_{sj} , \overline{C}_{sj} and \overline{T}_{sj} are diagonal, these new matrices are made with the diagonal elements as below:

$$a_{sj,i} = \begin{pmatrix} \frac{1}{r} + c \cdot hh_i + \frac{1}{\ell} \cdot hh'_i & 1 \\ 1 & 0 \end{pmatrix} \quad (54)$$

$$b_{sj,i} = \begin{pmatrix} c \cdot hl_i + \frac{1}{\ell} \cdot hl'_i & 0 \\ 0 & 0 \end{pmatrix} \quad (55)$$

$$c_{sj,i} = \begin{pmatrix} c \cdot lh_i + \frac{1}{\ell} \cdot lh'_i & 0 \\ 0 & 0 \end{pmatrix} \quad (56)$$

$$t_{sj,i} = \begin{pmatrix} \frac{1}{r} + c \cdot ll_i + \frac{1}{\ell} \cdot ll'_i & 1 \\ 1 & 0 \end{pmatrix} \quad (57)$$

(38) can be written as follow:

$$\left(C_R + pC_C + \frac{1}{p}C_L \right) \begin{pmatrix} v_1 \\ i_x \end{pmatrix} = \begin{pmatrix} 0 \\ v_{in} \end{pmatrix} \quad (58)$$

Where:

$$C_R = \begin{pmatrix} \frac{1}{r} & 1 \\ 1 & 0 \end{pmatrix}, \quad C_L = \begin{pmatrix} \frac{1}{\ell} & 0 \\ 0 & 0 \end{pmatrix}, \quad C_C = \begin{pmatrix} c & 0 \\ 0 & 0 \end{pmatrix} \quad (59)$$

C_R , C_C and C_L are respectively resistive coefficients matrix, capacitive coefficients matrix and inductive coefficients matrix. These matrices can be defined according to the modified nodal method. By applying (58) and (59), (54) to (57) are rearranged as follow:

$$a_{sj,i} = C_R + hh_i \cdot C_C + hh_i' \cdot C_L \quad (60)$$

$$b_{sj,i} = hl_i \cdot C_C + hl_i' \cdot C_L \quad (61)$$

$$c_{sj,i} = lh_i \cdot C_C + lh_i' \cdot C_L \quad (62)$$

$$t_{sj,i} = C_R + ll_i \cdot C_C + ll_i' \cdot C_L \quad (63)$$

The formulas (60) to (63) are written for any network directly. Now using these matrices the i^{th} value of response vectors are computed:

$$r_{sj,i} = t_{sj,i} - c_{sj,i} a_{sj,i}^{-1} b_{sj,i} \quad (64)$$

$$\hat{s}_{xj,i} = r_{sj,i}^{-1} (\hat{s}_{fj,i} - c_{sj,i} a_{sj,i}^{-1} \hat{d}_{fj,i}) \quad (65)$$

$$\hat{d}_{xj,i} = a_{sj,i}^{-1} (\hat{d}_{fj,i} - b_{sj,i} \hat{s}_{xj,i}) \quad (66)$$

To obtain j^{th} level of response vector in MRA space, i.e. d_{xj} and s_{xj} , Γ is multiplied to \hat{d}_{xj} and \hat{s}_{xj} respectively.

The steps for Nonsinusoidal steady state analysis are as follow:

1. Determine number of levels (J_{\max}) and number of samples (N), J_{\max} is the index of finest scale in multiresolution space.
2. Calculate the C_R , C_C and C_L matrices according to modified nodal method.
3. Set $J=1$.
4. Set $j=J_{\max}-J$, j is the index of current resolution level.
5. Compute D_T , $HD_T H$, $HD_T L$, ... for j^{th} level.
6. Compute the Γ matrix using (20) and then transfer the input vector to the new domain.
7. Set $i=1$.
8. Calculate $a_{sj,i}$, $b_{sj,i}$, $c_{sj,i}$ and $t_{sj,i}$ using (60) to (63). Then using (64) - (66) calculate $\hat{d}_{xj,i}$ and $\hat{s}_{xj,i}$.
9. If i is equal to $N/2^j$ then set $i=i+1$ and goto step 8.
10. Calculate the response vector in MRA space for j^{th} resolution level, (i.e. d_{xj} and s_{xj}).
11. If J is not equal to J_{\max} set $J=J+1$ and go to step 5.
12. End.

Fig. 3 shows the flowchart of nonsinusoidal steady state analysis in the proposed domain.

4. Case Study 1

In this section, the proposed method is demonstrated for a case study that is a test system for harmonic simulation

[26]. Fig. 4 shows a single line diagram of this network, a harmonic source where is located on the bus 49-RECT. For lines and transformers a simple model is used and the frequency dependency is neglected. The test system is connected to a larger plant from 100: Util-69 bus. So for the larger plant an equivalent is obtained from the fault MVA level.

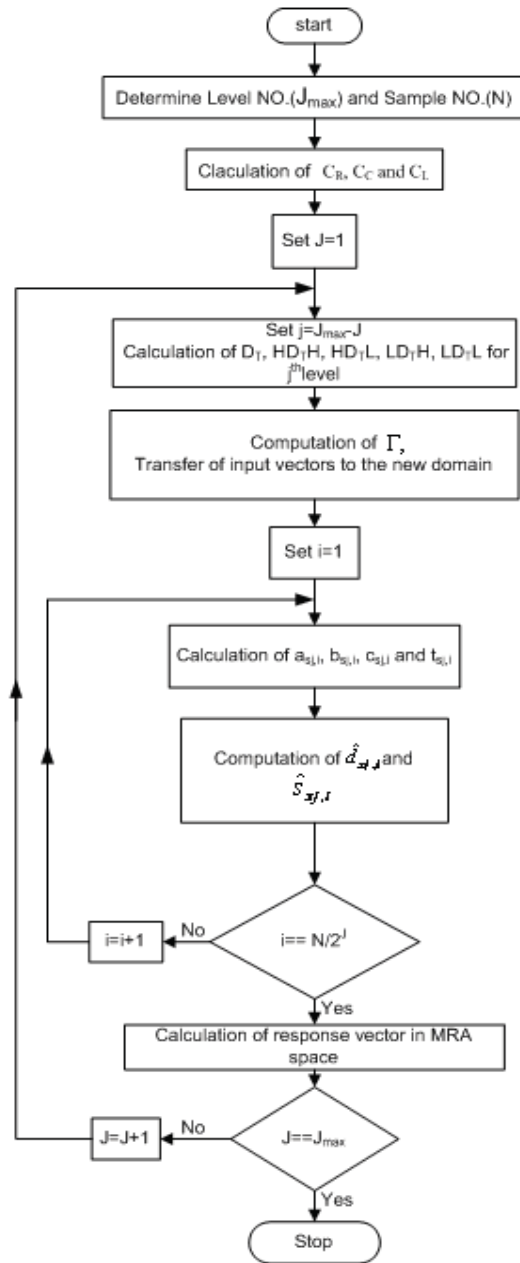


Fig. 3: Flowchart of nonsinusoidal steady state analysis.

The considered system consists of a local generator that is modeled using a voltage source with sub-transient

impedance. The system is completely modeled in the new suggested domain and the values of THDs are computed directly from results in this domain.. The consumed time for simulation of test network was found to be 4.7sec. Simulation, which was coded with MATLAB ver. 7, was carried out in this work using 512 samples, 3 levels for MRA space and using a personal computer with Pentium4

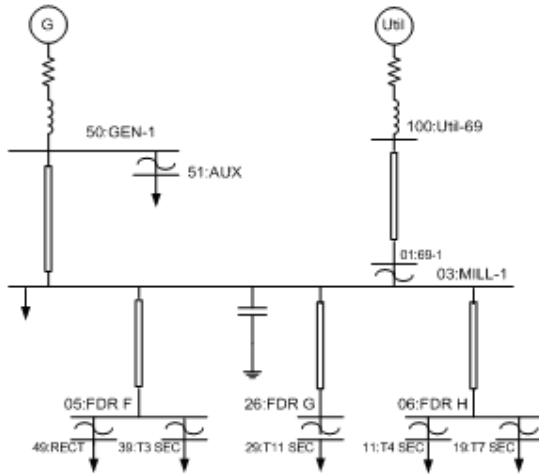


Fig. 4: The 13 bus test system

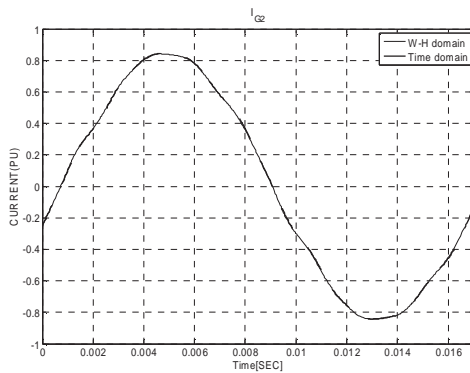


Fig. 5: Injected Current with Utility plant to test system, line: New domain, dotted-line: Time domain

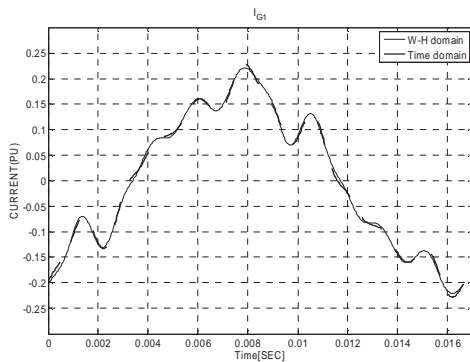


Fig. 6: Injected current with local generator (G1) to test system, line: New domain, dotted-line: Time domain.

CPU (2.8GHz) and 512RAM. To compare this approach with the time domain method, the currents of both the local generator and the utility plant are plotted in Fig. 5 and Fig. 6 respectively. In table I and II, some of the computed THDs are compared with those obtained from SIMULINK model using FFT.

Table 1: THD Percentage Values

Bus Voltages	New proposed method	Time domain method	Error (%)
49:RECT	2.48	2.57	3.568
05:FDR F	2.41	2.52	4.400
09:FDR H	2.40	2.52	4.789
03: MILL	2.41	2.52	4.396
50:GEN-1	2.32	2.43	4.497
100:UTIL-69	0.17	0.18	3.111

Table 2: THD Percentage Val UES

current	New proposed method	Time domain method	Error (%)
I_{UTIL}	2.49	2.54	2.106
I_{G1}	20.54	21.16	2.945

5. Case Study 2

To examine the validation of introduced models for distributed transmission line and switching devices a 132 KV single phase test system is selected and simulated via the new suggested domain. This simple system consists of three transmission lines and a switching load (Fig. 7). The switching load is a half bridge thyristor rectifier and it is supposed the firing angle is 5° . In table III test system parameters are presented. In Fig. 8 and Fig. 9 the results from time domain and new suggested domain simulations are compared. Also in table IV the THD percentages and the consumed times for this simulation are compared. The THDs are calculated directly from the new proposed domain simulation results with out transferring them to time domain. In this simulation db4 wavelet function is used and the time domain simulation is done with

MATLAB software. It must be noted that none of fast numerical methods such as sparse matrix methods are employed. However using fast numerical algorithms, especially in transferring input vectors and operators to the new domain, decrease the simulation time significantly.

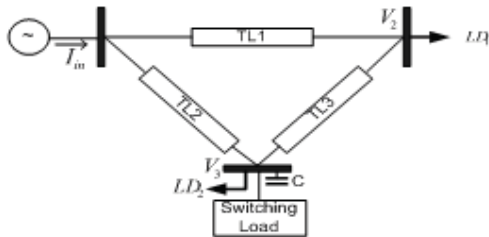


Fig. 7: Test system.

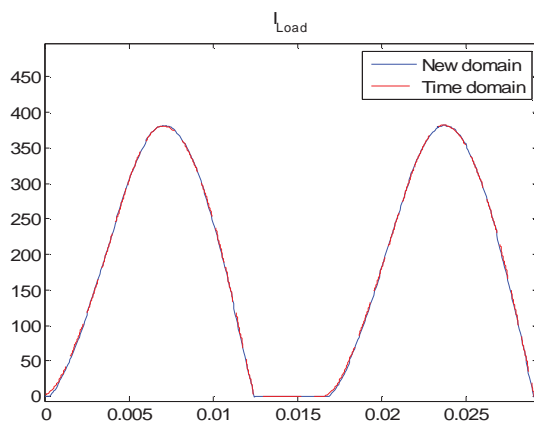


Fig. 8: Switching load current, line: New proposed domain; dash-line: Time domain

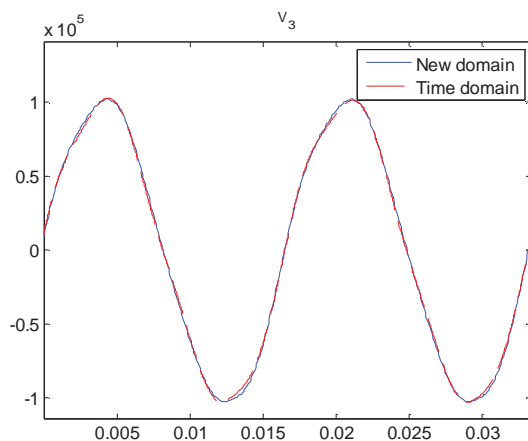


Fig. 9: V_3 , line: New proposed domain; dash-line: Time domain.

Table 3: Test System Parameters

$LD_1=LD_2=300+j267 [\Omega]$	$C=j377 [\Omega]$
$Length_{TL1}=100 [Km], Length_{TL2}=Length_{TL3}=50 [Km]$	
$R_{TL1}=R_{TL2}=R_{TL3}=0.0955 [\Omega/Km], L_{TL1}=L_{TL2}=L_{TL3}=2.137 \times 10^{-3} [H/Km]$	
$C_{TL1}=C_{TL2}=C_{TL3}=12.37 \times 10^{-9} [F/Km]$	
DC Load Impedance= $100+j377 [\Omega]$	

Table 4: Comparing Of THD Percentages and Simulation Times

	V_2	V_3	I_{IN}	SIMULATION TIME (SEC)
Time domain using FFT	3.4405	5.0874	65.259	4.31
New Domain (N=256, J=1)	3.3763	4.8023	65.617	1.41

5. Conclusion

In this paper a new algorithm for power system study and simulation in nonsinusoidal and periodic conditions via a new domain is introduced. The algorithm was applied to simulation of a relevant large test system containing a harmonic source. To show the ability of this domain for practical aims a single phase system with distributed transmission lines and a switching load was simulated. The model of switching device was based on switching signal. In such simulation the interaction between network and switching device was considered. The capabilities of this new method via new space make it possible to develop new applications in several fields such as power quality.

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