Robust Distributed Source Coding with Arbitrary Number of Encoders and Practical Code Design Technique

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Abstract:

The robustness property can be added to DSC system at the expense of reducing performance, i.e., increasing the sum-rate. The aim of designing robust DSC schemes is to trade off between system robustness and compression efficiency. In this paper, after deriving an inner bound on the rate–distortion region for the quadratic Gaussian MDC based RDSC system with two encoders, the structure of the RDSC system with three encoders and more generally with an arbitrary number of encoders are considered. Then inner bounds on the rate–distortion region for both MDC and MLC based Gaussian RDSC systems with an arbitrary number of encoders are derived. Finally, a practical coding approach for both MDC and MLC based Gaussian RDSC systems with an arbitrary number of encoders is proposed. The proposed approach is based on the multilevel Slepian-Wolf coded quantization framework. The approach is applied on the systems with two and three encoders and then extending and applying the approach on the systems with the number of encoders greater than three is straightforward. The obtained results are promising and satisfy the inner bounds for both rates and distortions for both sides and central decoders. This work paves the way of practical RDSC design in a general case.

Key words: Robust distributed source coding, Robust multiterminal source coding, Slepian-Wolf coded quantization

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1. Introduction

Distributed Source Coding (DSC) was introduced by Slepian and Wolf [1]. Multiterminal source coding (MTSC) [2], [3] is the generalization of the DSC problem to the separate lossy coding of two or more correlated sources. In one special case of MTSC, called indirect multiterminal source coding (IMTSC) or central executive officer (CEO) problem [4]-[7], each encoder cannot observe the source directly which is to be reconstructed at the decoder, but rather provides only a noisy version of the source. For finding the achievable rate region, due to the inherent difficulties of the general MTSC problem, researchers have focused on the quadratic Gaussian setup with Gaussian sources and the MSE distortion measure. For this special case, the achievable rate region was obtained for the IMTSC/CEO problem (with an arbitrary number of encoders) by Oohama [8] and Prabhakaran, et al. [9].

The main purpose of distributed source coding, i.e., Slepian-Wolf coding (SWC) and all of its various extensions, is compression efficiency. But the robustness of distributed source coding has not been considered as an intrinsic necessity. Consequently distributed source coding schemes are very sensitive to the encoder failure and any corruption (caused by packet loss or fading) in the transmitted codewords. The SW theorem was proved using random binning argument and the proof of achievability of rate–distortion region for the CEO problem (similar to the other extensions of SW coding problem) is also based on random binning argument [10], [11]. The main idea of random binning, practically implemented via algebraic binning and channel coding principles, is to randomly partition all codewords into disjoint bins. In this case, instead of directly sending a codeword, the index of the bin containing that codeword is transmitted. After receiving the indices of bins from all encoders, the decoder picks a codeword from each bin such that these codewords are jointly typical. Obviously if the decoder has access to the data from one of the encoders, then the correct codeword can not be recovered since the decoder only gets the index of a bin from one encoder where this bin contains many codewords.

The robustness property can be added to DSC system at the expense of reducing performance, i.e., increasing the sum-rate. Since the number of bins determines the compression efficiency and the number of codewords determines the description ability (robustness property) of the encoder, if the size of bin is small enough, the robustness of the distributed source coding system can be improved. With such a robust DSC scheme, if one encoder fails during the transmission, the decoder will be able to recover the data with a lower fidelity.

By combining the random binning technique and some ideas from robust source coding techniques such as multiple description coding (MDC) and multiple layers coding (MLC) the trade off between system robustness and compression efficiency can be possible. In such a robust distributed source coding (RDSC) system, each of encoders sends coded versions or descriptions of its observed signal. If descriptions (at each encoder) are obtained in MDC sense, i.e. the information content of descriptions are similar, we call the system as MDC based RDSC. If descriptions (at each encoder) are obtained in the MLC sense, i.e. the first description (layer) is a coarser version of the second description (layer), we call the system as MLC based RDSC.

1.1. Related Works

For the RDSC problem, only a few theoretical works have been done and no practical coding schemes, based on algebraic binning and channel coding principles, have been proposed. Indeed, researches on the theoretical aspects of RDSC and practical code design techniques for it, are still in their starting stages.

Ishwar *et al.* [12] introduced a RDSC scheme for IMTSC/CEO problem by increasing the sum-rate and deriving a new rate limit. Such an RDSC scheme would be able to recover the source by receiving bitstreams from any K out of N encoders. If more than K bitstreams are received, the quality will be more improved [12]. Such an RDSC scheme is very similar to N-channel symmetric MDC [13]. The main difference between RDSC scheme of [12] and N-channel symmetric MDC scheme of [13] is that each encoder's observation is generated independently in [12].

For IMTSC/CEO setup with two encoders, Chen and Berger [14] theoretically studied a RDSC scheme. In such a scheme, each of two encoders sends two coded versions or descriptions of its observed signal. For the quadratic Gaussian MLC based RDSC system with two encoders, an inner bound on the rate–distortion region was derived in [14]. However in [14], differences between MDC and MLC based RDSC systems are not clarified.

In [23] a coding scheme is proposed for RDSC based on only source coding methodology. As mentioned in [23], the channel coding and source coding methodologies to address DSC and RDSC will actually solve substantially different problems; hence the goals of the coding method of [23] and the coding method of our work in this paper are different.

1.2. Comparison with Related Works

In this paper, first differences between MDC and MLC based RDSC systems are clarified and an inner bound on the rate–distortion region for the quadratic Gaussian MDC based RDSC system with two encoders is derived. Then the structure of the RDSC system with an arbitrary number of encoders is considered. Inner bounds on the rate–distortion region for both MDC and MLC based Gaussian RDSC system with arbitrary number of encoders are derived. Also, for the first time, we propose a practical coding approach for both MDC and MLC based Gaussian RDSC systems with an arbitrary number of encoders. The proposed approach is based on the multilevel Slepian- Wolf coded quantization framework. We apply the approach on the RDSC systems with two and three encoders. Applying the approach on the systems with the number of encoders greater than three is straightforward. The obtained results are promising and satisfy the inner bounds for both rates and distortions for both sides and central decoders. We think that this work paves the way of practical RDSC design in a general sense. In summary, the main contributions of this paper are:

1) Clarification between MDC based RDSC system and MLC based RDSC system;

2) Derivation of an inner bound on the rate–distortion region for MDC based RDSC system in Gaussian case;

3) Extension of the RDSC system to scenarios of RDSC systems with an arbitrary number of encoders in both MLC and MDC based cases and also the derivation of inner bounds on the rate–distortion region for them;

4) Practical code design, for the first time, for RDSC systems with two and three encoders in both MLC and MDC based cases with promising results.

The rest of the paper is organized as follows: in section 2, RDSC system for two encoders is studied. Then for quadratic Gaussian case, the theoretical inner bound for both MDC and MLC based systems are discussed. Section 3 introduces scenarios of RDSC systems with an arbitrary number of encoders, also the corresponding inner bounds on the rate–distortion region are derived in this section. In section 4, practical approach based on Slepian-Wolf coded quantization is proposed. The result of our simulations based on proposed approach for RDSC problem using both MDC and MLC techniques at the encoders are presented in section 5. Section 6 concludes the paper.

2. RDSC Problem with Two Encoders

Fig. 1 shows RDSC system model which was used by Chen and Berger [14]. Each of two encoders sends two coded versions (descriptions at general sense) of its observed signal. Each side decoder can only receive and decode the first description which is sent by only one of the encoders, but the third decoder (the central decoder) can receive all four descriptions sent by both encoders.

If two descriptions (from each encoder) are obtained in MDC sense, i.e. the information content of both descriptions are similar, we call the system as MDC based RDSC system. If two descriptions are obtained in the MLC sense, i.e. the first description (layer) is a coarser version of the second description (layer), we call the system as MLC based RDSC system.

For RDSC setup with two observations, there are two encoders and each of them has two encoding functions

$$f_{l,i} : \Upsilon_l^n \to \{1,2,\ldots,2^{nR_{l,i}}\} \;,\; l=1,2 \quad i=1,2. \tag{1}$$

And also there are three decoders with the decoder functions as:

$$g_1 : \{1,2,\ldots,2^{nR_{1,1}}\} = C_{1,1}^n \to X^n$$
$$g_2 : \{1,2,\ldots,2^{nR_{2,1}}\} = C_{2,1}^n \to X^n$$
$$g_{12} : \{1,2,\ldots,2^{nR_{1,1}}\} \times \{1,2,\ldots,2^{nR_{1,2}}\} \times \{1,2,\ldots,2^{nR_{2,1}}\} \times \{1,2,\ldots,2^{nR_{2,2}}\} = C_{1,1}^n \times C_{1,2}^n \times C_{2,1}^n \times C_{2,2}^n \to X^n \tag{2}$$

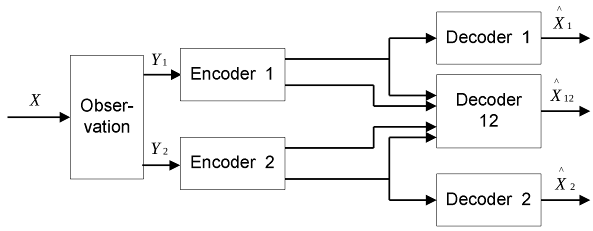

Fig. 1: RDSC system based on Chen-Berger model [14].

In this setup, the 5-tuples $(R_1, R_2, D_1, D_2, D_{12})$ is achievable if there are auxiliary random variables $U_{1,1}, U_{1,2}, U_{2,1}$ and $U_{2,2}$ (these auxiliary random variables are interpreted as quantized versions of Y_1 and Y_2) such that they form the following Markov chains:

$$(U_{1,1}, U_{1,2}) \to Y_1 \to (X, Y_2, U_{2,1}, U_{2,2})$$

and $(U_{2,1}, U_{2,2}) \to Y_2 \to (X, Y_1, U_{1,1}, U_{1,2})$

The Chen-Berger rate region, R_{CB}, with respect to distortions D_1, D_2 and D_{12} is defined as [14]:

$$R_{CB}(D_1, D_2, D_{12}) = conv\left(\bigcup_{U(D_1,D_2,D_{12})} R(U_{1,1}, U_{1,2}, U_{2,1}, U_{2,2}) \right) \tag{3}$$

where

$$R(U_{1,1},U_{1,2},U_{2,1},U_{2,2}) = \{(R_1,R_2) : R_1 \geq I(Y_1;U_{1,1}) + I(Y_1;U_{1,2} \mid U_{1,1},U_{2,1},U_{2,2}),$$
$$R_2 \geq I(Y_2;U_{2,1}) + I(Y_2;U_{2,2} \mid U_{2,1},U_{1,1},U_{1,2}),$$
$$R_1 + R_2 \geq I(Y_1;U_{1,1}) + I(Y_2;U_{2,1}) + I(Y_1,Y_2;U_{1,2},U_{2,2} \mid U_{1,1},U_{2,1})\} \tag{4}$$

and $U(D_1, D_2, D_{12})$ is the set of all $(U_{1,1}, U_{1,2}, U_{2,1}, U_{2,2})$ where there exist the decoder functions g_1, g_2 and g_{12} of (4) such that $\hat{X}_1 = g_1(U_{1,1})$, $\hat{X}_2 = g_2(U_{2,1})$,

$\hat{X}_{12} = g_{12}(U_{1,1}, U_{1,2}, U_{2,1}, U_{2,2})$ and

$$\frac{1}{n}\sum_{t=1}^{n} E\left[d\left(X(t), \hat{X}_p(t) \right)\right] \le D_p + \varepsilon \quad p = 1, 2 and 12 \qquad (5)$$

One can show that the rate bounds of (6) can be rewritten as follows:

$$R_1 \ge H(U_{1,1}) + H(U_{1,2} \mid U_{1,1}, U_{2,1}, U_{2,2}),$$

$$R_2 \ge H(U_{2,1}) + H(U_{2,2} \mid U_{2,1}, U_{1,1}, U_{1,2})$$

$$R_1 + R_2 \ge H(U_{1,1}) + H(U_{2,1}) + H(U_{1,2}, U_{2,2} \mid U_{1,1}, U_{2,1}) . \qquad (6)$$

It is easy to verify that for MLC based RDSC, we can assume that $U_{l,1}$ and $U_{l,2}$ form the Markov chain $U_{l,1} \to U_{l,2} \to Y_l$, $l = 1,2$, and g_{12} can be defined only with input $\{1,2,...,2^{nR_{1,2}}\} \times \{1,2,...,2^{nR_{2,2}}\}$.

2.1. Quadratic Gaussian Case

The results of previous section which were obtained for the finite alphabet case with bounded distortion measure can be extended to the Gaussian case with squared distortion measure [14]. Hence we will generalize the case for the quadratic Gaussian CEO problem. In the quadratic Gaussian IMTSC/CEO problem, $\{X(t)\}_{t=1}^{\infty}$ is a sequence of i.i.d. Gaussian random variables with zero mean and variance σ_X^2 , $\{Y_1(t) = X(t) + N_1(t)\}_{t=1}^{\infty}$ and $\{Y_2(t) = X(t) + N_2(t)\}_{t=1}^{\infty}$ are sequences of i.i.d. zero mean Gaussian random variables and $X(t)$, $N_1(t)$ and $N_2(t)$ are mutually independent. Variance of $N_l(t)$ is $\sigma_{N_l}^2$ for $l = 1,2$.

For the robust quadratic Gaussian IMTSC/CEO problem, auxiliary random variables $U_{1,1}, U_{1,2}, U_{2,1}$ and $U_{2,2}$ ar defined as follows:

$$U_{l,i} = Y_l + Q_{l,i} = X + N_l + Q_{l,i}$$

$$l = 1,2, \text{ and } i = 1,2.$$

where $Q_{l,i}$ (quantization noise) is zero mean Gaussian random variable with variance $\sigma_{Q_{l,i}}^2$. $Q_{1,1}, Q_{1,2}, Q_{2,1}$ and $Q_{2,2}$ are independent of X, Y_1 and Y_2 ; also $Q_{1,1}, Q_{1,2}$ are independent of $Q_{2,1}, Q_{2,2}$.

For the case that each encoder of the robust IMTSC system uses MLC, i.e. the quantization associated to $U_{l,1}$ is the coarser version of the quantization associated to $U_{l,2}$, the following inner bound of rate-distortion region was derived in [14] based on the achievability of Chen-Berger rate region :

$$C_{CB\_IB} = conv\left(\bigcup_{\sigma^2_{Q_{1,1}} \ge \sigma^2_{Q_{1,2}}, \sigma^2_{Q_{2,1}} \ge \sigma^2_{Q_{2,2}}} C(\sigma^2_{Q_{1,1}}, \sigma^2_{Q_{1,2}}, \sigma^2_{Q_{2,1}}, \sigma^2_{Q_{2,2}}) \right) \qquad (7)$$

where

$$C(\sigma^2_{Q_{1,1}}, \sigma^2_{Q_{1,2}}, \sigma^2_{Q_{2,1}}, \sigma^2_{Q_{2,2}}) = \left\{ \begin{array}{l} (R_1, R_2, D_1, D_2, D_{12}) : D_l \ge \left(\frac{1}{\sigma_X^2} + \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,1}}} \right)^{-1}, l = 1,2, \\ D_{12} \ge \left(\frac{1}{\sigma_X^2} + \sum_{l=1}^{2} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,2}}} \right)^{-1}, \\ \text{and } R_1, R_2 \text{ satisfy the conditions of (6)} \end{array} \right\} \qquad (8)$$

For the case that each encoder of the RDSC system uses MDC, at the central decoder after reconstruction of $U_{l,1}$ and $U_{l,2}$, these two descriptions are combined and signal $U_{l,0,2}$ is obtained which yields a better estimate for Y_l . It is assumed that $U_{l,0,2}$, *l=1, 2,* are zero mean Gaussian random variables with variance $\sigma^2_{Q_{l,0,2}}$. For the MDC based RDSC, the following inner bound is introduced in this paper based on the achievability of Chen-Berger rate region:

$$C_{IB\_MDC\_2} = conv\left(\bigcup_{\sigma^2_{Q_{1,0,2}}, \sigma^2_{Q_{1,1}}, \sigma^2_{Q_{1,2}}, \sigma^2_{Q_{2,0,2}} \sigma^2_{Q_{2,1}}, \sigma^2_{Q_{2,2}}} C^*(\sigma^2_{Q_{1,0}}, \sigma^2_{Q_{1,1}}, \sigma^2_{Q_{1,2}}, \sigma^2_{Q_{2,0}} \sigma^2_{Q_{2,1}}, \sigma^2_{Q_{2,2}}) \right) \qquad (9)$$

where

$$C^*(\sigma^2_{Q_{1,0,2}}, \sigma^2_{Q_{1,1}}, \sigma^2_{Q_{1,2}}, \sigma^2_{Q_{2,0,2}} \sigma^2_{Q_{2,1}}, \sigma^2_{Q_{2,2}}) = \left\{ \begin{array}{l} \{(R_1, R_2, D_1, D_2, D_{12}) : D_l \ge \left(\frac{1}{\sigma_X^2} + \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,1}}} \right)^{-1}, l = 1,2; \\ D_{12} \ge \left(\frac{1}{\sigma_X^2} + \sum_{l=1}^{2} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,0,2}}} \right)^{-1}, \text{and } R_1, R_2 \text{ satisfy the conditions of (8)} \end{array} \right\} \qquad (10)$$

Note that the above expressions for distortions in (8) and (10), are the infinite-rate bounds of the distortions [15]. These bounds are the minimum achievable values for distortions and are corresponding to the case when the sum-rate tends to infinity.

3. RDSC Problem with More than Two Encoders

3.1. RDSC Problem with Three Encoders

Fig. 2 shows the proposed RDSC system with three encoders. Each of three encoders sends three coded versions (descriptions at general sense) of its observed signal. Each of three decoders (Decoder 1, Decoder 2 and Decoder 3) can only receive and decode the first description which is sent by only one of the encoders, each of other three decoders (Decoder 12, Decoder 13 and Decoder 23) receive and decode two descriptions sent by two encoders and the central decoder can receive all descriptions sent by all three encoders. Like the RDSC system with two encoders if descriptions (in each encoder) are obtained in the MDC sense, the system is called as MDC based RDSC system and if descriptions (in each encoder) are obtained in the MLC sense the system is MLC based RDSC system.

Encoding functions, $f_{l,i}$ $l=1,2,3$ and $i=1,2,3$, and decoding functions, g_p, $p=1,2,3,12,13,23,123$, are as follows:

$$f_{l,i}:\mathcal{Y}_l^n \rightarrow \{1,2,\ldots,2^{nR_{l,i}}\}=C_{l,i}^n \quad , \quad l=1,2,3 \text{ and } i=1,2,3.$$

$$g_1:\{1,2,\ldots,2^{nR_{1,1}}\}=C_{1,1}^n \rightarrow \mathcal{X}^n$$

$$g_2:\{1,2,\ldots,2^{nR_{2,1}}\}=C_{2,1}^n \rightarrow \mathcal{X}^n$$

$$g_3:\{1,2,\ldots,2^{nR_{3,1}}\}=C_{3,1}^n \rightarrow \mathcal{X}^n$$

$$g_{12}:\{1,2,\ldots,2^{nR_{1,1}}\}\times\{1,2,\ldots,2^{nR_{1,2}}\}\times\{1,2,\ldots,2^{nR_{2,1}}\}\times\{1,2,\ldots,2^{nR_{2,2}}\}=C_{1,1}^n\times C_{1,2}^n\times C_{2,1}^n\times C_{2,2}^n \rightarrow X^n$$

$$g_{13}:\{1,2,\ldots,2^{nR_{1,1}}\}\times\{1,2,\ldots,2^{nR_{1,2}}\}\times\{1,2,\ldots,2^{nR_{3,1}}\}\times\{1,2,\ldots,2^{nR_{3,2}}\}=C_{1,1}^n\times C_{1,2}^n\times C_{3,1}^n\times C_{3,2}^n \rightarrow X^n$$

$$g_{23}:\{1,2,\ldots,2^{nR_{2,1}}\}\times\{1,2,\ldots,2^{nR_{2,2}}\}\times\{1,2,\ldots,2^{nR_{3,1}}\}\times\{1,2,\ldots,2^{nR_{3,2}}\}=C_{2,1}^n\times C_{2,2}^n\times C_{3,1}^n\times C_{3,2}^n \rightarrow X^n$$

$$g_{123}:\{1,2,\ldots,2^{nR_{1,1}}\}\times\{1,2,\ldots,2^{nR_{1,2}}\}\times\{1,2,\ldots,2^{nR_{1,3}}\}\{1,2,\ldots,2^{nR_{2,1}}\}\times\{1,2,\ldots,2^{nR_{2,2}}\}\times\{1,2,\ldots,2^{nR_{2,3}}\}\times\{1,2,\ldots,2^{nR_{3,1}}\}\times\{1,2,\ldots,2^{nR_{3,2}}\}\times\{1,2,\ldots,2^{nR_{3,3}}\}=$$
$$C_{1,1}^n\times C_{1,2}^n\times C_{1,3}^n\times C_{2,1}^n\times C_{2,2}^n\times C_{2,3}^n\times C_{3,1}^n\times C_{3,2}^n\times C_{3,3}^n \rightarrow X^n \tag{11}$$

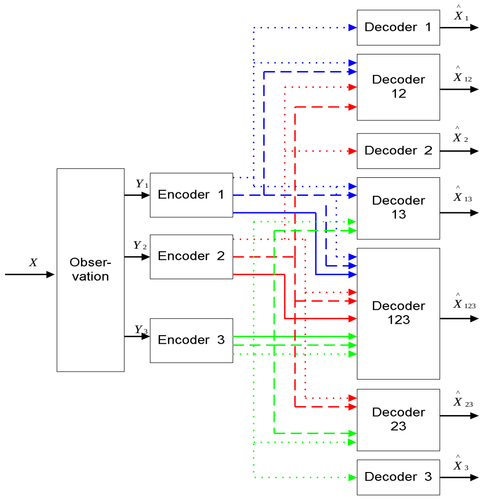

Fig. 2: RDSC system model with three encoders.

For easier representation, we define the following sets:

$U=\{U_{l,i} \mid l=1,2,3$ and $i=1,2,3\}$ and $Y=\{Y_1,Y_2,Y_3\}$.

The following theorem determines an achievable rate – distortion region.

Theorem1.

$(R_1,R_2,R_3,D_1,D_2,D_3,D_{12},D_{13},D_{23},D_{123})$ is achievable if there exist auxiliary random variables $U_{l,i}$, $l=1,2,3$ and $i=1,2,3$, such that:

1) They form the following Markov chains:

$$(U_{l,1},U_{l,2},U_{l,3}) \rightarrow Y_l \rightarrow (X, Y\setminus Y_l, U\setminus\{U_{l,1},U_{l,2},U_{l,3}\})$$
$l=1,2,3$.

2) $(R_1,R_2,R_3)\in R(U)$ where

$$R(U)=\{(R_1,R_2,R_3):$$
$$R_l \geq I(Y_l;U_{l,1})+\sum_{k\in I_3\setminus\{l\}} I(Y_l;U_{l,2}\mid U_{l,1},U_{k,1},U_{k,2})+I(Y_l;U_{l,3}\mid U\setminus U_{l,3}) \quad l\in I_3$$
$$R_l+R_k \geq I(Y_l;U_{l,1})+I(Y_k;U_{k,1})+I(Y_l,Y_k;U_{l,2},U_{k,2}\mid U_{l,1},U_{k,1})$$
$$+I(Y_l;U_{l,2}\mid U_{l,1},U_{m,1},U_{m,2})+$$
$$I(Y_k;U_{k,2}\mid U_{k,1},U_{m,1},U_{m,2})+I(Y_l;U_{l,3}\mid U\setminus\{U_{l,3}\}) \quad k,l,m\in I_3 \quad k\neq m\neq l$$
$$+I(Y_k;U_{k,3}\mid U\setminus\{U_{k,3}\})$$
$$R_1+R_2+R_3 \geq \sum_{l\in I_3} I(Y_l;U_{l,1})+\sum_{l,k\in I_3} \begin{matrix} I(Y_l,Y_k;U_{l,2},U_{k,2}\mid U_{l,1},U_{k,1}) \\ +I(Y;U_{1,3},U_{2,3},U_{3,3}\mid U\setminus\{U_{1,3},U_{2,3},U_{3,3}\}) \end{matrix}\} \tag{12}$$

3) There exist decoder functions of (13) such that:

$$\hat{X}_1=g_1(U_{1,1}),\ \hat{X}_2=g_2(U_{2,1}),\ \hat{X}_3=g_3(U_{3,1}),$$
$$\hat{X}_{12}=g_{12}(U_{1,1},U_{1,2},U_{2,1},U_{2,2}),$$
$$\hat{X}_{13}=g_{13}(U_{1,1},U_{1,2},U_{3,1},U_{3,2}),$$
$$\hat{X}_{23}=g_{23}(U_{3,1},U_{3,2},U_{2,1},U_{2,2}),\ \hat{X}_{123}=g_{123}(U) \tag{13}$$

and

$$\frac{1}{n}\sum_{t=1}^{n} E\left[d\left(X(t),\hat{X}_p(t)\right)\right]\leq D_p+\varepsilon, \quad for\ p=1,2,3,12,13,23,123. \tag{14}$$

Proof. See Appendices A.

One can show that the rate bounds of (12) can be rewritten as follows

$$R_l \geq H(U_{l,1})+\sum_{k\in I_3\setminus\{l\}} H(U_{l,2}\mid U_{l,1},U_{k,1},U_{k,2})+H(U_{l,3}\mid U\setminus U_{l,3}) \quad l\in I_3$$

$$
\begin{aligned}
R_l + R_k &\ge H(U_{l,1}) + H(U_{k,1}) \\
&+ H(U_{l,2}, U_{k,2} \mid U_{l,1}, U_{k,1}) \\
&+ H(U_{l,3} \mid U_{l,1}, U_{m,1}, U_{m,2}) + \\
&H(U_{k,3} \mid U_{k,1}, U_{m,1}, U_{m,2}) \\
&+ H(U_{l,3} \mid U \setminus \{U_{l,3}\}) \qquad k,l,m \in I_3 \quad k \neq m \neq l \\
&+ H(U_{k,3} \mid U \setminus \{U_{k,3}\})
\end{aligned}
$$

.

$$
\begin{aligned}
R_1 + R_2 + R_3 &\ge \sum_{l \in I_3} H(U_{l,1}) \\
&+ \sum_{l,k \in I_3} H(U_{l,2}, U_{k,2} \mid U_{l,1}, U_{k,1}) + H(U_{1,3}, U_{2,3}, U_{3,3} \mid U \setminus \{U_{1,3}, U_{2,3}, U_{3,3}\})
\end{aligned}
\tag{15}
$$

It is easy to verify that for MLC based RDSC, we can assume that $U_{l,1}, U_{l,2}$ and $U_{l,3}$ form the Markov chain $U_{l,1} \to U_{l,2} \to U_{l,3} \to Y_l$.

In the Gaussian setup, similar to the case with two encoders, observation noises, N_l, $l = 1,2,3$, quantization noises, $Q_{l,i}$, and auxiliary random variables $U_{l,i}$ (these auxiliary random variables are interpreted as quantized versions of Y_l) are defined and we have

$U_{l,i} = Y_l + Q_{l,i} = X + N_l + Q_{l,i}$ for $l = 1,2,3$ and $i = 1,2,3.$

For the case that each encoder of the RDSC system uses MLC, the following inner bound of rate-distortion region is derived based on the achievability of the rate region of Theorem 1:

$$
C_{IB\_MLC3} = conv\left(\bigcup_{\sigma^2_{Q_{1,1}} \ge \sigma^2_{Q_{1,2}} \ge \sigma^2_{Q_{1,3}}, \sigma^2_{Q_{2,1}} \ge \sigma^2_{Q_{2,2}} \ge \sigma^2_{Q_{2,3}}, \sigma^2_{Q_{3,1}} \ge \sigma^2_{Q_{3,2}} \ge \sigma^2_{Q_{3,3}}} C(\sigma^2_{Q_{1,1}}, \sigma^2_{Q_{1,2}}, \sigma^2_{Q_{1,3}}, \sigma^2_{Q_{2,1}}, \sigma^2_{Q_{2,2}}, \sigma^2_{Q_{2,3}}, \sigma^2_{Q_{3,1}}, \sigma^2_{Q_{3,2}}, \sigma^2_{Q_{3,3}}) \right) \tag{16}
$$

where

$$
C(\sigma^2_{Q_{1,1}}, \sigma^2_{Q_{1,2}}, \sigma^2_{Q_{1,3}}, \sigma^2_{Q_{2,1}}, \sigma^2_{Q_{2,2}}, \sigma^2_{Q_{2,3}}, \sigma^2_{Q_{3,1}}, \sigma^2_{Q_{3,2}}, \sigma^2_{Q_{3,3}}) = \left\{ \begin{array}{l}
(R_1, R_2, R_3, D): \\
D_l \ge \left(\frac{1}{\sigma^2_X} + \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,1}}} \right)^{-1}, l = 1,2,3 \\
D_{12} \ge \left(\frac{1}{\sigma^2_X} + \sum_{l=1,2} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,2}}} \right)^{-1}, D_{13} \ge \left(\frac{1}{\sigma^2_X} + \sum_{l=1,3} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,2}}} \right)^{-1}, \\
D_{23} \ge \left(\frac{1}{\sigma^2_X} + \sum_{l=2,3} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,2}}} \right)^{-1}, D_{123} \ge \left(\frac{1}{\sigma^2_X} + \sum_{l=1}^{3} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,3}}} \right)^{-1} \\
\text{and } R_1, R_2, R_3 \text{ satisfy the conditions of (15)}
\end{array} \right\} \tag{17}
$$

For the case that each encoder of the RDSC system uses MDC, at the central decoder (decoder 123) after reconstruction of $U_{l,1}, U_{l,2}$ and $U_{l,3}$, $l = 1,2,3,$ these three descriptions of Y_l are combined and signal $U_{l,0,3}$ is obtained. Also at the decoder 12, after reconstruction of $U_{l,1}$ and $U_{l,2}$, $l = 1,2,$ these two descriptions of Y_l are combined and signals $U_{l,0,2}$, $l = 1,2,$ are obtained. Similarly signals $U_{l,0,2}$, $l = 1,3,$ are obtained at decoder 13 and signals $U_{l,0,2}$, $l = 2,3,$ are obtained at decoder 23. It is assumed that signals $U_{l,0,3}$ and $U_{l,0,2}$, $l = 1,2,3,$ are i.i.d. zero mean Gaussian random variables with variances $\sigma^2_{Q_{l,0,3}}$ and $\sigma^2_{Q_{l,0,2}}$. For the MDC based RDSC with three encoders, the following inner bound of rate-distortion region is derived based on the achievability of the rate region of Theorem 1:

$$
C_{IB\_MDC\_3} = conv\left(\bigcup_{\{\sigma^2_Q\}} C(\{\sigma^2_Q\}) \right) \quad \text{where}
$$

$\{\sigma^2_Q\} = \{\sigma^2_{Q_{l,i}}, \sigma^2_{Q_{l,0,K}}\}$, $l \in I_3, i \in I_3$, $K = 2,3$, and

$$
C(\{\sigma^2_Q\}) = \left\{ \begin{array}{l}
(R_1, R_2, R_3, D): \\
D_l \ge \left(\frac{1}{\sigma^2_X} + \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,1}}} \right)^{-1}, l = 1,2,3 \\
D_{12} \ge \left(\frac{1}{\sigma^2_X} + \sum_{l=1,2} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,0,2}}} \right)^{-1}, D_{13} \ge \left(\frac{1}{\sigma^2_X} + \sum_{l=1,3} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,0,2}}} \right)^{-1}, \\
D_{23} \ge \left(\frac{1}{\sigma^2_X} + \sum_{l=2,3} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,0,2}}} \right)^{-1}, D_{123} \ge \left(\frac{1}{\sigma^2_X} + \sum_{l=1}^{3} \frac{1}{\sigma^2_{N_l} + \sigma^2_{Q_{l,0,3}}} \right)^{-1}, \\
\text{and } R_1, R_2, R_3 \text{ satisfy the conditions of (15)}
\end{array} \right\} \tag{18}
$$

3.2. RDSC Problem with Arbitrary Number of Encoders

The setup of RDSC system with L encoders is the generalization of the setup of RDSC system with three encoders. Each encoder sends L coded versions (descriptions at general sense) of its observed signal. There are $2^L - 1$ decoders. The central decoder can receive all descriptions sent by all encoders. Each of L side decoders can only receive and decode the first description which is sent by each encoder, and the other decoders receive and decode a fraction of descriptions sent by encoders. In general, there are L^2 encoding functions, $f_{l,i}$, $l \in I_L$ and $i \in I_L$, and $2^L - 1$ decoding functions, $g_x \in G$, where

$$G = \left\{ g_{p_1}, g_{p_1 p_2}, g_{p_1 p_2 p_3}, \ldots g_{p_1 p_2 \ldots p_L} \;\middle|\; p_l \in I_L \quad l \in I_L \quad p_1 < p_2 < \ldots < p_L \right\}$$
.

Note that $|G| = 2^L - 1$ where |.| denotes cardinality of a set.

For easier representation, we define the following sets

$U = \{U_{l,i} \mid l = 1,2,\ldots,L$ and $i = 1,2,\ldots,L\}$,

$Y = \{Y_1, Y_2, \ldots, Y_L\}$ and

$$D = \left\{ D_{p_1}, D_{p_1 p_2}, D_{p_1 p_2 p_3}, \ldots D_{p_1 p_2 \ldots p_L} \;\middle|\; p_l \in I_L \quad l \in I_L \quad p_1 < p_2 < \ldots < p_L \right\}$$
.

Theorem 2. $(R_1, R_2, \ldots, R_L, D)$ is achievable if there are auxiliary random variables $U_{l,i}$, $l = 1,2,\ldots,L$ and $i = 1,2,\ldots,L$, such that:

1) They form the following Markov chains:
$$(U_{l,1}, U_{l,2}, \ldots, U_{l,L}) \to Y_l \to (X, Y \setminus Y_l, U \setminus \{U_{l,1}, U_{l,2}, \ldots, U_{l,L}\})$$
$l = 1,2,\ldots,L$.

2) $(R_1, R_2, \ldots, R_L) \in R(U)$ where,

$$\begin{aligned}
&R(U) = \{(R_1, R_2, \ldots, R_L):\\
&R_l \ge I(Y_l; U_{l,1}) + \sum_{i \ge 2} I(Y_l; U_{l,i} \mid U_{l,I_i \setminus \{i\}}, U_{j_1,I_i}, \ldots U_{j_{i-1},I_i}) \quad l \in I_L \quad j_n \in I_L \setminus \{l, j_1, \ldots, j_{n-1}\} \quad n = 1,\ldots,i-1,\\
&R_l + R_k \ge I(Y_l; U_{l,1}) + I(Y_k; U_{k,1}) + I(Y_l, Y_k; U_{l,2}, U_{k,2} \mid U_{l,1}, U_{k,1}) + \sum_{i \ge 2} I(Y_l; U_{l,i} \mid U_{l,I_i \setminus \{i\}}, U_{j_1,I_i}, \ldots U_{j_{i-1},I_i})\\
&+ \sum_{i \ge 2} I(Y_k; U_{k,i} \mid U_{k,I_i \setminus \{i\}}, U_{j_1,I_i}, \ldots U_{j_{i-1},I_i}) \quad l,k \in I_L \quad j_n \in I_L \setminus \{l, k, j_1, \ldots, j_{n-1}\} \quad n = 1,\ldots,i-1,\\
&.\\
&.\\
&.\\
&R_1 + R_2 + \ldots + R_L \ge \sum_{l \in I_L} I(Y_l; U_{l,1}) + \sum_{l_1 \in I_L} \sum_{l_2 > l_1} I(Y_{l_1}, Y_{l_2}; U_{l_1,2}, U_{l_2,2} \mid U_{l_1,1}, U_{l_2,1}) + \ldots\\
&+ \sum_{l_1 \in I_L} \sum_{l_2 > l_1} \ldots \sum_{l_K > l_{K-1}} I(Y_{l_1}, Y_{l_2}, \ldots Y_{l_K}; U_{l_1,K}, U_{l_2,K}, \ldots U_{l_K,K} \mid U_{l_1,I_{K-1}}, U_{l_2,I_{K-1}}, \ldots U_{l_K,I_{K-1}}) + \ldots\\
&+ I(Y_{l_1}, Y_{l_2}, \ldots Y_{l_L}; U_{l_1,L}, U_{l_2,L}, \ldots U_{l_L,L} \mid U_{l_1,I_{L-1}}, U_{l_2,I_{L-1}}, \ldots U_{l_L,I_{L-1}})\}
\end{aligned} \tag{19}$$

3) There exist decoder functions $g_x \in G$ such that

$\hat{X}_x = g_x(V)$, $V \subseteq U$ and

$$\frac{1}{n} \sum_{t=1}^{n} E\left[d\left(X(t), \hat{X}_x(t) \right) \right] \le D_x + \varepsilon \quad D_x \in D. \tag{20}$$

Proof. See Appendices B.

It is easy to verify that for MLC based RDSC, the Markov chain $U_{l,1} \to U_{l,2} \to \ldots \to U_{l,L} \to Y_l$ is formed.

In the Gaussian setup, for the case of L encoders, similar to the cases with two or three encoders observation noises, N_l, quantization noises, $Q_{l,i}$, and auxiliary random variables $U_{l,i}$ (these auxiliary random variables are interpreted as quantized versions of Y_l) are defined.

For MLC based RDSC, the following inner bound of rate-distortion region is derived based on the achievability of the rate region of Theorem 2:

$$C_{IB\_MLC\_L} = conv\left(\bigcup_{\sigma^2_{Q_{l,1}} \ge \sigma^2_{Q_{l,2}} \ge \ldots \ge \sigma^2_{Q_{l,L}}, \forall l \in I_L} C(\sigma^2_{Q_{l,i}}) \right)$$
$$l, i \in I_L \tag{21}$$

where,

$$C(\sigma^2_{Q_{l,i}}) = \left\{ \begin{aligned}
&(R_1, R_2, \ldots, R_L, D):\\
&D_{p_1} \ge \left(\frac{1}{\sigma_X^2} + \frac{1}{\sigma_{N_l}^2 + \sigma_{Q_{l,1}}^2} \right)^{-1}, l = p_1 \in I_L\\
&D_{p_1 p_2} \ge \left(\frac{1}{\sigma_X^2} + \sum_{i = p_1, p_2} \frac{1}{\sigma_{N_i}^2 + \sigma_{Q_{i,2}}^2} \right)^{-1}, p_1, p_2 \in I_L, p_2 > p_1\\
&D_{p_1 p_2 p_3} \ge \left(\frac{1}{\sigma_X^2} + \sum_{i = p_1, p_2, p_3} \frac{1}{\sigma_{N_i}^2 + \sigma_{Q_{i,3}}^2} \right)^{-1}, p_1, p_2, p_3 \in I_L, p_3 > p_2 > p_1\\
&.\\
&.\\
&.\\
&D_{p_1 p_2 \ldots p_L} \ge \left(\frac{1}{\sigma_X^2} + \sum_{i = p_1, \ldots, p_L} \frac{1}{\sigma_{N_i}^2 + \sigma_{Q_{i,L}}^2} \right)^{-1}, p_1, \ldots, p_L \in I_L, p_L > \ldots > p_2 > p_1\\
&\text{and } R_1, R_2, \ldots, R_L \text{ satisfy the conditions of (19).}
\end{aligned} \right\} \tag{22}$$

For the case that each encoder of the RDSC system uses MDC, at the decoder labeled by $p_1 p_2 \ldots p_K$ after reconstruction of $U_{l,1}, U_{l,2} \ldots U_{l,K}$, $l = p_1, p_2, \ldots, p_K$, $p_1, \ldots, p_K \in I_L$, $p_K > \ldots > p_2 > p_1$, $K \le L$, these descriptions of Y_l are combined and signals $U_{l,0,K}$ are obtained. It is assumed that signals $U_{l,0,K}$, are i.i.d. zero mean Gaussian random variables with variances $\sigma^2_{Q_{l,0,K}}$. For the MDC based RDSC with L encoders, the following inner bound of rate-distortion region is derived based on the achievability of the rate region of Theorem 2:

$$C_{IB\_MDC\_L} = conv\left(\bigcup_{\{\sigma_Q^2\}} C(\{\sigma_Q^2\}) \right) \tag{23}$$

where

$$\{\sigma_Q^2\} = \{\sigma^2_{Q_{l,i}}, \sigma^2_{Q_{l,0,K}}\} \qquad l \in I_L, i \in I_L, \; K \in I_L \setminus \{1\}$$

$$
C(\{\sigma_Q^2\}) = \left\{ \begin{array}{l} (R_1, R_2, \ldots, R_L, D): \\ D_{p_1} \geq \left(\frac{1}{\sigma_X^2} + \frac{1}{\sigma_{N_l}^2 + \sigma_{Q_{l,1}}^2} \right)^{-1}, l = p_1 \in I_L \\ D_{p_1 p_2} \geq \left(\frac{1}{\sigma_X^2} + \sum_{l=p_1,p_2} \frac{1}{\sigma_{N_l}^2 + \sigma_{Q_{l,0,2}}^2} \right)^{-1}, p_1, p_2 \in I_L, p_2 > p_1 \\ D_{p_1 p_2 p_3} \geq \left(\frac{1}{\sigma_X^2} + \sum_{l=p_1,p_2,p_3} \frac{1}{\sigma_{N_l}^2 + \sigma_{Q_{l,0,3}}^2} \right)^{-1}, p_1, p_2, p_3 \in I_L, p_3 > p_2 > p_1 \\ . \\ . \\ . \\ D_{p_1 p_2 p_3 \ldots p_L} \geq \left(\frac{1}{\sigma_X^2} + \sum_{l=p_1,p_2,\ldots p_L} \frac{1}{\sigma_{N_l}^2 + \sigma_{Q_{l,0,L}}^2} \right)^{-1}, p_1, p_2, \ldots, p_L \in I_L, p_L > \ldots > p_2 > p_1 \\ \text{and } \mathrm{R}_1, \mathrm{R}_2, \ldots, \mathrm{R}_L \text{ satisfy the conditions of (19)} \end{array} \right\} \quad (24)
$$

4. Proposed Coding Method

It can be inferred from the theoretical works of [8], [9] that efficient quantization and SWC (binning) is optimal for the quadratic Gaussian MTSC with two encoders. This result is used in [22] for code design for MTSC problem with two encoders. Recently Tavildar, Viswanath and Wagner [24] have shown that natural analog digital separation can be optimal for code lossy DSC problems with arbitrary number of encoders. In our previous works of [25], [26] and [27] we use this framework for code design for IMTSC/CEO with arbitrary number of encoders and multimedia multicasting based on IMTSC.

The main idea of RDSC using Slepian-Wolf-Cover coded quantization is as follows: several descriptions are formed and binned separately at the l^{th} encoder; the bin index of each description is sent to the decoder, and the decoder recovers the descriptions of all the encoders successively according to a prescribed decoding order. The coding scheme associated with RDSC using Slepian-Wolf-Cover coded quantization was depicted in Fig. 3.

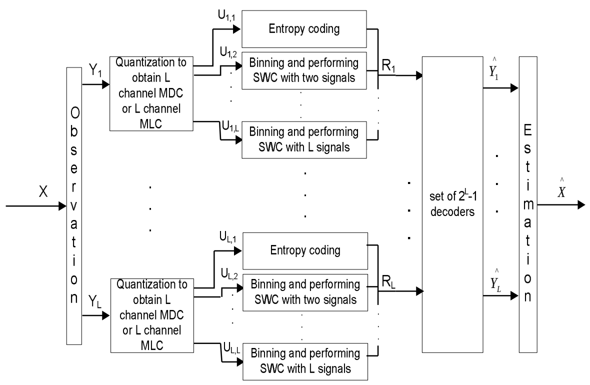

Fig. 3: RDSC using Slepian-Wolf-Cover coded quantization framework

The approach is applied on the RDSC systems with two and three encoders. Extending and applying the approach on the RDSC systems with the number of encoders greater than three is straightforward.

4.1 MLC versus MDC at Encoders

In this paper, MDC and MLC techniques are implemented via proper scalar quantization approaches. In MLC case, at each encoder, MLC can be implemented by nested scalar quantizers; the input of all L quantizers at l^{th} encoder is Y_l and the output of the quantizers from the finest quantizer to the coarsest one are associated with $U_{l,L}$ to $U_{l,1}$, respectively. In MDC case, L staggered quantizers which have similar performance are used to produce the quantized signals $U_{l,i}$, $i = 1,2,\ldots L$. As mentioned before, at the central decoder after reconstruction of $U_{l,i}, i = 1,2,\ldots L$, these descriptions are combined to obtain signals $U_{l,0,L}$. In fact signals $U_{l,0,L}, l = 1,2,\ldots,L,$, can be produced by a quantizer which is a combination of all quantizers which produce $U_{l,i}$, $i = 1,2,\ldots L$. The quantization schemes for the corresponding signals, for MDC and MLC based RDSC are shown in Fig. 4 and Fig. 5, respectively, with two and three encoders.

The problem of designing MDC by scalar quantization is called multiple description scalar quantization (MDSQ) and involves two steps: quantization of the source, and solving of the index assignment problem which is a mapping of an integer source to a tuple to be transmitted [16]. The index assignment problem for MDSQ with three or more descriptions is a complex mathematical problem and has connections with theory of graph bandwidth [17].

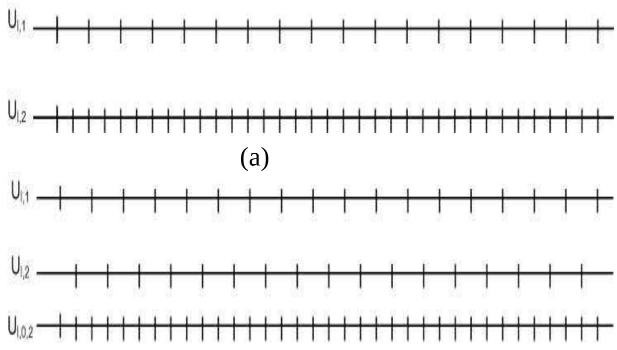

Fig. 4: Quantization schemes and the corresponding signals for: (a) MLC based RDSC system with two encoders, and (b) MDC based RDSC system with two encoders.

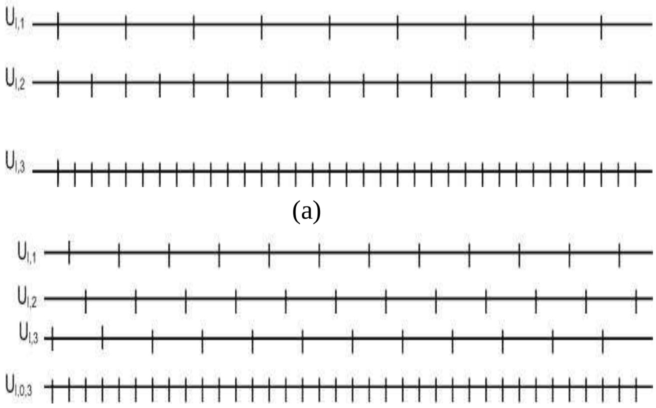

Fig. 5: Quantization schemes and the corresponding signals for: (a) MLC based RDSC system with three encoders, and (b) MDC based RDSC system with three encoders.

Regarding the lower bounds for sum-rates in (6) and (15), we attempt to achieve $H(U_{l,1})$, $H(U_{l,2},U_{k,2}|U_{l,1},U_{k,1})$ and $H(U_{1,3},U_{2,3},U_{3,3}|U\setminus\{U_{1,3},U_{2,3},U_{3,3}\})$. The n-length observed source vector Y_l^n is quantized to obtain $U_{l,i}^n$, for $l\in I_L$ and $i\in I_L$. We assume that $U_{l,i}$ is obtained by using m_l - bit dithered scalar quantizer. Obviously for MLC based RDSC, $m_L\geq...\geq m_2\geq m_1$, and for MDC based RDSC , $m_L=...=m_2=m_1$. $U_{l,i}$ can be written in terms of its bit-plane representation. In particular $U_{l,1},U_{l,2}$ and $U_{l,3}$ are written as $U_{l,1}=(b_{l1}b_{l2}...b_{lm_1})$, $U_{l,2}=(c_{l1}c_{l2}...c_{lm_2})$ and $U_{l,3}=(d_{l1}d_{l2}...d_{lm_3})$. The *first* bit plane, b_{l1}, c_{l1} or d_{l1}, represents the least significant bit plane and the *last* bit plane, b_{lm_1}, c_{lm_2} or d_{lm_3} , represents the most significant bit plane. In a RDSC system with L encoders, achieving to $H(U_{1,L},U_{2,L},...,U_{L,L}|U\setminus\{U_{1,L},U_{2,L},...,U_{L,L}\})$ is possible by performing SWC with L signals. Similar method can be considered for coding order to perform SWC with K , $K\leq L$, signals to achieve any constituent term of the sum-rate formula in (19).

4.2. Conditional Multilevel Slepian-Wolf Coding

The conditional entropy of $H(U_{l,2},U_{k,2}|U_{l,1},U_{k,1})$ can be written as

$$H(U_{l,2},U_{k,2}|U_{l,1},U_{k,1})=H(c_{l1},c_{l2},...c_{lm_2},c_{k1},c_{k2},...c_{km_2}|b_{l1},b_{l2},...b_{lm_1},b_{k1},b_{k2},...b_{km_1})$$
$$=\sum_{i=1}^{m_2}H(c_{li},c_{ki}|M_{i-1}^2) \tag{25}$$

where $M_{i-1}^2=(U_{l,1},U_{k,1},c_{l1},c_{l2},...,c_{l(i-1)},c_{k1},c_{k2},...,c_{k(i-1)})$. By using the chain rule, an expression for $H(c_{li},c_{ki}|M_{i-1}^2)$ is

$$H(c_{li},c_{ki}|M_{i-1}^2)=H(c_{li}|M_{i-1}^2)+H(c_{ki}|c_{li},M_{i-1}^2). \tag{26}$$

Similarly, the conditional entropy of $H(U_{1,3},U_{2,3},U_{3,3}|U_{1,1},U_{2,1},U_{3,1},U_{1,2},U_{2,2},U_{3,2})$ can be written as

$$H(U_{1,3},U_{2,3},U_{3,3}|U_{1,1},U_{2,1},U_{3,1},U_{1,2},U_{2,2},U_{3,2})$$
$$=H(d_{11},d_{12},...d_{1m_3},d_{21},d_{22},...d_{2m_3},d_{31},d_{32},...d_{3m_3}|U_{1,1},U_{2,1},U_{3,1},U_{1,2},U_{2,2},U_{3,2})$$
$$=\sum_{i=1}^{m_3}H(d_{1i},d_{2i},d_{3i}|M_{i-1}^3)=H(d_{1i}|M_{i-1}^3)+H(d_{2i}|d_{1i},M_{i-1}^3)+H(d_{3i}|d_{1i},d_{2i},M_{i-1}^3) \tag{27}$$

where, $M_{i-1}^3=(U_{1,1},U_{2,1},U_{3,1},U_{1,2},U_{2,2},U_{3,2},d_{11},d_{12},...,d_{1(i-1)},$ $d_{21},d_{22},...,d_{2(i-1)},d_{31},d_{32},...,d_{3(i-1)})$.

In (26) and (27), the net information of the i^{th} bit plane of one description of one of the sources after considering all kinds of existing correlations, i.e. correlation among sources due to correlated observations, correlation among descriptions of one source and cross bit plane correlation for different bit planes of one description, is obtained.

Multilevel SWC must be capable of compressing the quantized sources to their joint entropy. Constructive approach to achieve SW bounds is the algebraic binning which is implemented via channel coding ideas (syndrome based or parity based approaches) [19]. LDPC codes, due to their capacity approaching property, are the most advanced channel codes used for the implementing of the SWC [18], [19]. But there are some gaps between the theoretical bounds (conditional entropies) and the corresponding practical compression rates achieved by LDPC codes.

In fact density evolution is an algorithm for computing the capacity of low-density parity-check (LDPC) codes under message passing decoding [20]. Density evolution is described and analyzed in detail in [20]. For memory-less binary-input continuous-output additive white Gaussian noise (AWGN) channels and message passing decoders, a Gaussian approximation is used for message densities under density evolution to simplify the analysis of the decoding algorithm. Further explanation about Gaussian approximations was given in [21]. References [20] and [21] are cited by almost all works that use LDPC codes.

For the BSC model, the BS channel can be viewed as an AWGN channel with the quantized output. In an approximate method to design the LDPC code for the BSC model, BSC parameters are first mapped to that of the AWGN channel and then the process of Gaussian approximation is used for the equivalent AWGN channel. The mapping is based on the equality of the stability functions for the two channels [19]:

$$\left(\frac{E_s}{N_0}\right)_{eq} = -\log\left(2\sqrt{p(1-p)}\right) \tag{28}$$

where p is the crossover probability of the BSC and $(E_s / N_0)_{eq}$ is the equivalent AWGN channel parameter.

This equation can be used for mapping of BSC correlation channel (between two binary correlated sources X and Y) with crossover probability of p to its equivalent AWGN correlation channel [19]. Assuming asymmetric syndrome based SWC (compression Y to H(Y) and X to H(X|Y)), it can be shown that H(X|Y) = H(p). For this problem an LDPC code with rate $\frac{k}{n}$ is required so that:

$$\frac{n-k}{n} \geq H(X \mid Y) = H(p) = -p\log p - (1-p)\log(1-p) \tag{29}$$

For the system with two encoders, regarding the syndrome based SWC and the framework of multilevel coding, to achieve $H(c_{1i}, c_{2i} \mid M_{i-1}^2)$, LDPC codes $\mathbb{C}_{i,1,2}$ *(n, $k_{i,1,2}$)* and $\mathbb{C}_{i,2,2}$ *(n, $k_{i,2,2}$)* with rates $(\frac{k_{i,1,2}}{n})$ and $(\frac{k_{i,2,2}}{n})$ are used. The rates of these codes are selected such that $\frac{n-k_{i,1,2}}{n} \geq H(c_{1i} \mid M_{i-1}^2)$ and $\frac{n-k_{i,2,2}}{n} \geq H(c_{2i} \mid c_{1i}, M_{i-1}^2)$. Then, syndrome based SWC is performed on c_{1i} and c_{2i} using $\mathbb{C}_{i,1,2}$ *(n, $k_{i,1,2}$)* and $\mathbb{C}_{i,2,2}$ *(n, $k_{i,2,2}$)*. For the system with three encoders, to achieve $H(d_{1i}, d_{2i}, d_{3i} \mid M_{i-1}^3)$, LDPC codes $\mathbb{C}_{i,1,3}$ *(n, $k_{i,1,3}$)*, $\mathbb{C}_{i,2,3}$ *(n, $k_{i,2,3}$)* and $\mathbb{C}_{i,3,3}$ *(n, $k_{i,3,3}$)* with rates $(\frac{k_{i,1,3}}{n})$, $(\frac{k_{i,2,3}}{n})$ and $(\frac{k_{i,3,3}}{n})$ are used. The rates of these codes are selected such that $\frac{n-k_{i,1,3}}{n} \geq H(d_{1i} \mid M_{i-1}^3)$, $\frac{n-k_{i,2,3}}{n} \geq H(d_{2i} \mid d_{1i}, M_{i-1}^3)$ and $\frac{n-k_{i,3,3}}{n} \geq H(d_{3i} \mid d_{1i}, d_{2i}, M_{i-1}^3)$. Then, syndrome based SWC is performed on d_{1i}, d_{2i} and d_{3i} using $\mathbb{C}_{i,1,3}$ *(n, $k_{i,1,3}$)*, $\mathbb{C}_{i,2,3}$ *(n, $k_{i,2,3}$)* and $\mathbb{C}_{i,3,3}$ *(n, $k_{i,3,3}$)*, respectively.

5. Simulation Results

In this section, the results of some of our experiments on RDSC problem by using two separate cases of MDC and MLC techniques are given. In all of the following experiments, the remote source and observation noises are zero mean Gaussian and mutually independent random variables. The conditional entropies are computed using Monte Carlo simulations.

5.1. MLC and MDC based RDSC with Two Encoders

The remote source with variance $\sigma_x^2 = 1$ is assumed. For MLC case at each encoder, two nested scalar quantizers are employed to obtain $U_{l,1}, U_{l,2}$, with 3 and 4 bits per symbol, respectively. For MDC case at each encoder, two staggered scalar quantizers are employed to obtain $U_{l,1}, U_{l,2}$, with 3 bits per symbol. The corresponding value for mean square distortion between $U_{l,i}$ and Y_l is denoted by ζ_{li} and reported in Table 1. According to (6), the theoretical inner bound for sum-rate is $R_1 + R_2 = H(U_{1,1}) + H(U_{2,1}) + H(U_{1,2}, U_{22} \mid U_{1,1}, U_{2,1})$. To achieve $H(U_{1,1})$ and $H(U_{2,1})$, adaptive arithmetic coding is employed separately at each encoder to exploit cross bit plane correlations. The results of applying adaptive arithmetic coding compared with $H(U_{1,1})$ and $H(U_{2,1})$ have a gap of only 0.0015 bit per symbol. The conditional entropies and the appropriate transmission rates for the various bit planes of $U_{1,2}$ and $U_{2,2}$ are given in Table 1. According to the theoretical inner bound for the sum-rate, the total rate losses are 0.0305 and 0.0459 bit per symbol for MLC and MDC case respectively. The inner bound or the infinite-rate bound (the minimum achievable value) of the distortions for the side and the central decoders, according to (8) and (10), along with the practical obtained distortions are given in Table 1. As Table 1 shows, all practically obtained distortion values satisfy the corresponding inner bounds. In particular, the gap between the practical and the minimum achievable values for distortion in the central decoder is only about 0.0143 and 0.0008 for MLC and MDC case respectively.

Table 1: Theoretical and practical characteristics of MDC and MLC based RDSC systems with two encoders.

Type of RDSC	MLC	MDC
Observation noise levels	$\sigma_{N_1}^2 = \sigma_{N_2}^2 = 0.01$	$\sigma_{N_1}^2 = \sigma_{N_2}^2 =$ 0.02
mean square distortion between Y_i and $U_{l,i}$ denoted by ζ_{li}	ζ_{11}= 0.3394, ζ_{12}= 0.0834, ζ_{21}= 0.3404 and ζ_{22}= 0.0836	ζ_{11}= 0.0567, ζ_{12}= 0.0647, ζ_{21}= 0.0554 and ζ_{22}= 0.0536
mean square distortions between $U_{l,0,2}$ and Y_i	-	ζ_{102}= 0.0164 , ζ_{202}= 0.0145
Theoretical sum rate bound : R_{sum} (bits per symbol)	3.6612	6.2914

$H(U_{1,1})$	1.2818	2.4026
Practical rate corresponding to $H(U_{1,1})$	1.2833	2.4041
$H(U_{2,1})$	1.2789	2.4822
Practical rate corresponding to $H(U_{2,1})$	1.2804	2.4832
$H(U_{1,2},U_{2,2}\|U_{1,1},U_{2,1})$	1.1005	1.4067
Practical rate corresponding to $H(U_{1,2},U_{2,2}\|U_{1,1},U_{2,1})$	1.128	1.45

Bit plane	1	2	3	4	Sum	1	2	3	4	Sum
$H\ (c_{1i}\ \|$	0.0062	0.4346	0.3415	0	0.7823	0.1807	0.1711	0.3853	-	0.7371
Practical rate: $1-(k_{i,1,2}$	0.007	0.44	0.35	0	0.797	0.19	0.18	0.39	-	0.76
$H\ (c_{2i}\ \|$	0.0005	0.1506	0.1626	0	0.3182	0.1759	0.1618	0.3318	-	0.6695
Practical rate: $1-(k_{i,2,2}$	0.006	0.155	0.17	0	0.331	0.185	0.17	0.335	-	0.69

Decoder	Side (1)	Central (12)	Side (2)	Side (1)	Central (12)	Side (2)
The minimum achievable distortion	0.2589	0.0447	0.2597	0.0712	0.0174	0.0701
The practically obtained distortion	0.3501	0.059	0.3508	0.0757	0.0182	0.0741

5.2. MLC and MLC Based RDSC with Three Encoders

The remote source with variance $\sigma_x^2=1$ is assumed. For MLC case at each encoder, three nested scalar quantizers are employed are employed to obtain $U_{l,1}, U_{l,2}$ and $U_{l,3}$ with 3, 5 and 8 levels, respectively. For MDC case at each encoder, three staggered scalar quantizers, with the same number of bits per symbol, are employed to obtain $U_{l,1}, U_{l,2}$ and $U_{l,3}$. The corresponding value for mean square distortion between $U_{l,i}$ and Y_l is denoted by ζ_{li} and reported in Table 2. For MDC case The corresponding values for mean square distortions between $U_{l,0,3}$ and Y_l, denoted by ζ_{103}, is reported in Table 2. According to (15), the theoretical inner bound for sum-rate is

$$R_1+R_2+R_3\geq\sum_{l\in I_3}H(U_{l,1})$$
$$+\sum_{l,k\in I_3}H(U_{l,2},U_{k,2}|U_{l,1},U_{k,1})+H(U_{1,3},U_{2,3},U_{3,3}|U\setminus\{U_{1,3},U_{2,3},U_{3,3}\})$$

. To achieve the entropies $H(U_{1,1})$, $H(U_{2,1})$ and $H(U_{3,1})$ adaptive arithmetic coding is employed separately at each encoder to exploit cross bit plane correlations. Three similar procedures, similar to that one described in Table 1, are performed to achieve $H(U_{1,2},U_{2,2}|U_{1,1},U_{2,1})$, $H(U_{1,2},U_{3,2}|U_{1,1},U_{3,1})$ and $H(U_{2,2},U_{3,2}|U_{2,1},U_{3,1})$. The corresponding rate losses (gaps) are reported in Table 2. To achieve $H(U_{1,3},U_{2,3},U_{3,3}|U\setminus\{U_{1,3},U_{2,3},U_{3,3}\})$, the conditional multilevel Slepian-Wolf coding with three encoders is applied. Conditional entropies and appropriate transmission rates for the various bit planes of $U_{1,2}$, $U_{2,2}$ and $U_{3,3}$ are given in Table 2. According to the theoretical inner bound for the sum-rate, the total rate losses are 0.138 and 0.176 bit per symbol for MLC and MDC cases respectively. The inner bound or the infinite-rate bound (the minimum achievable value) of the distortions for three side decoders (each side decoder receives only one description from only one of encoders) and the central decoder, according to (17) and (18), along with the practically obtained distortions are given in Table 2. As Table 2 shows, all practically obtained distortion values satisfy the inner bounds.

Table 2: Theoretical and practical characteristics of MDC and MLC based RDSC systems with three encoders.

Type of RDSC	MLC	MDC
Observation noise levels	$\sigma_{N_1}^2=\sigma_{N_2}^2=\sigma_{N_3}^2$ 0.02	$\sigma_{N_1}^2=\sigma_{N_2}^2=\sigma_{N_3}^2$ 0.02
mean square distortion between Y_i and $U_{l,i}$ denoted by ζ_{li}	ζ_{11}= 2.162, ζ_{12}= 0.4222, ζ_{13}= 0.1064, ζ_{21}= 2.2239, ζ_{22}= 0.4245, ζ_{23}= 0.1077, ζ_{31}= 2.3585, ζ_{32}= 0.4281 and ζ_{33}= *0.1065*.	ζ_{11}= 0.2164, ζ_{12}= 0.213, ζ_{13}= 0.2256, ζ_{21}= 0.2114, ζ_{22}= 0.2132, ζ_{23}= 0.2257, ζ_{31}= 0.2171, ζ_{32}= 0.2063 and ζ_{33}= 0.2271
mean square distortions between $U_{l,0,3}$ and Y_i	-	ζ_{103}= 0.0277 , ζ_{203}= 0.0277 and ζ_{303}= 0.0279
Theoretical sum rate bound : R_{sum} (bits per symbol)	5.1508	9.3568
Practical sum rate	5.2886	9.5328
$\sum_{l\in I_3} H(U_{l,1})$ (Theoretical)	0.823+0.8564+0.9347	1.554+1.5509+1.5545
Practical rates corresponding to $\sum_{l\in I_3} H(U_{l,1})$	0.8245+0.8579+0.9362	1.5554+1.5524+1.556
$\sum_{l,k\in I_3} H(U_{l,2},U_{k,2} \mid U_{l,1},U_{k,1})$ (Theoretical)	0.4014+0.3796+0.354	1.2099+1.1964+1.1995
Practical rates corresponding to $\sum_{l,k\in I_3} H(U_{l,2},U_{k,2} \mid U_{l,1},U_{k,1})$	0.435 +0.405 +0.38	1.235 +1.22 +1.225
$H(U_{1,3},U_{2,3},U_{3,3} \mid U\backslash\{U_{1,3},U_{2,3},U$ (Theoretical)	1.4018	1.0917
Practical rates corresponding to $H(U_{1,3},U_{2,3},U_{3,3} \mid U\backslash\{U_{1,3},U_{2,3},U_{3,}$	1.45	1.19

Bit plane	1	2	3	Sum	1	2	3	Sum
$H(d_{1i} \mid M_{i-1}^3)$	0.5755	0.1581	0	0.7336	0.2352	0.2112	0	0.4464
Practical rate: $1-(k_{i,1,3}/n)$	0.585	0.165	0	0.75	0.245	0.23	0	0.475
$H(d_{2i} \mid d_{1i}, M_{i-}^3$	0.2589	0.0882	0	0.3471	0.1738	0.1695	0	0.3433
Practical rate: $1-(k_{i,2,3}/n)$	0.265	0.1	0	0.365	0.19	0.185	0	0.375
$H(d_{3i} \mid d_{1i}, d_{2i},$	0.2151	0.106	0	0.3211	0.159	0.143	0	0.302
Practical rate: $1-(k_{i,3,3}/n)$	0.22	0.115	0	0.335	0.175	0.165	0	0.34
Decoder	Central (123)	Side (1)	Side (2)	Side (3)	Central (123)	Side (1)	Side (2)	Side (3)
The minimum achievable distortion	0.0406	0.6857	0.6917	0.704	0.0157	0.1912	0.1879	0.1917
The practically obtained distortion	0.054	1.2013	1.2579	1.4022	0.0198	0.2384	0.2327	0.2362

6. Conclusion

In this paper, considering the main view point about RDSC, the structure of the RDSC system with an arbitrary number of encoders are considered and inner bounds on the rate–distortion region for both MDC and MLC based Gaussian RDSC systems with an arbitrary number of encoders are derived.

The practical coding approach which used in this paper is based on the multilevel Slepian-Wolf coded quantization framework. The results of applying the approach satisfy the inner bounds for both rates and distortions for both MDC and MLC based Gaussian RDSC systems.

Our focus in this paper was on the theoretical limits of the RDSC system with an arbitrary number of encoders and an implementation with a moderate complexity. Future works can include improving results by using stronger source and channel codes. In future works other functionalities can be added to the system. By using distributed successive refinement principles, at least for the quadratic Gaussian CEO problem which can be implemented by successive Wyner–Ziv coding schemes, scalable RDSC systems are achievable. Scalable RDSC seems to be especially attractive in wireless sensor networks, where channels are subject to fluctuations.

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